KONINKLIJKE NEDERLANDSCHE AKADEMIE VAN WETENSCHAPPEN

PROCEEDINGS

VOLUME LII

No. 8

President: A. J. KLUYVER
Secretary: M. W. WOERDEMAN

1949
NORTH-HOLLAND PUBLISHING COMPANY
(N.V. Noord-Hollandsche Uitgevers Mij.)
AMSTERDAM

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(Communicated at the meeting of September 24, 1949.)

1. *Introduction.*

In KRUYT's Colloid Science Vol. II one of us (B. de J.) has given a survey of the peculiar phenomena observed with drops of phosphatide coacervates [1]).

These phenomena, viz. *a*) abnormal behaviour of the vacuoles, *b*) abnormal behaviour of the drops in a d.c. field, c) occurrence of multiple coacervate drops, had since their discovery [2]) not been observed in coacervates of other colloids, neither in those of macromolecular colloids (e.g. gelatin) nor in those of association colloids (e.g. soaps). This isolated position of phosphatide coacervates has now come to an end as we have learned to realize coacervates of soaps showing quite the same phenomena.

In the following this new type of soap coacervates will be called *P-coacervates* (coacervates with phosphatide-like proporties), to distinguish them from the "ordinary" soap coacervates (lacking these properties), which will be called *O-coacervates.*

2. *Morphology of drops of O- and P-coacervates of oleates, including their behaviour in a d.c. field.*

For the present investigation, which deals only with oleate coacervates, we started from "Sodium Oleate, Neutral, Powder" from BAKER, which preparation, though not representing really pure Na-oleate, was the only one accesible to us in the after wartime [3]). As its impurities — mainly the soaps of the C_{16} and C_{18} saturated fatty acids — may show P-coacervation also under similar conditions (though not at numerically the same), this admixture will not radically change the results which may be expected from chemically pure oleate.

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[1]) H. R. KRUYT, Colloid Science Vol. II, Elsevier Publishing Company, Inc. New York, Amsterdam, London, Brussels 1949. Compare Chapter XI, § 6.

[2]) H. G. BUNGENBERG DE JONG and R. F. WESTERKAMP. Biochem. Z. 248 335 (1932).

[3]) The first indications for the occurrence of P-coacervates were found during the war starting from Na oldinicum med. pur. pulv. MERCK. As it was impossible to buy this preparation anew, we applied to the ROCKEFELLER Foundation, which kindly put at our disposal a large quantity of the preparation mentioned in the text in order to enable us to continue our researches on oleate systems. We are glad to express our great thanks for his help.

As we know already that oleates behave with regard to O-coacervation in an uncomplicated way only, if the pH is high enough (e.g. at pH 12 and higher) to exclude all hydrolysis [4]), it seemed advisable to start our study on P-coacervation under the same precaution. As far as we know at present P-coacervation of oleates can be obtained only by the combined action of a salt (e.g. KCl, K_2CO_3) and a typical polar organic non-electrolyte of not too small molecular weight (e.g. isoamylalcohol, n.hexanol, n.heptanol, cyclohexanol, terpineol, benzylalcohol, phenylaethylalcohol). For the present investigation our choice fell on the combination KCl + isoamylalcohol, $(CH_3)_2CH\text{-}CH_2\text{-}CH_2OH$, a purissimum preparation of MERCK being used for all experiments.

If we now prepare a dilute oleate solution (e.g. 2 %) containing 0.1 N. KOH (to ensure a pH above 12) and add 10 ml 2N.KCL to 10 ml of it, we obtain after rigorously shaking a homogeneous and (apart from entrapped air bubbles) a clear solution which is very highly viscous and which shows marked elastic proporties as well. Starting from this one phase system we can readely obtain O-coacervation and P-coacervation by adding different amounts of isoamylalcohol and rigorously shaking (for the experimental equipment actually used see section 4). A few small drops will suffice already to start *O. coacervation.* The total system now consists of two phases: the O coacervate and the equilibrium liquid. If we inspect the system under the microscope (for technique see section 4) we will see the coacervate in the form of perfectly round drops [5]).

When two of such drops meet, they readely fuse. If vacuoles are present in the coacervate drops, their behaviour is quite normal (when two vacuoles meet, they readily fuse; a vacuole meeting the innerside of the drop boundary breaks though and the drop boundary rounds off). Also in a d.c. field we do not perceive the peculiar behaviour of the vacuoles which we will meet with the P-coacervate. The main effect is electrophoresis of the drops, which show a negative charge at their surface.

If we gradually add now more and more isoamylalcohol, the total oleate system in the tube changes its character (for details of the various successive states of the oleate system see section 4) and when a fairly large number of drops is added, it becomes homogeneous and clear again. The obtained one phase system now needs only one or two drops of isoamylalcohol to start coacervation afresh, this time the *P coacervate* separating. Microscopical inspection at once reveals an unexpected morphological richdom:

1) Many coacervate drops are not perfectly round, but have the shape of spheres

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[4]) At lower pH (e.g. at 10 already) complications occur (the oleic acid formed by hydrolysis causing "auto sensibilisation") and below pH 8.70 coacervation is no longer possible at all by KCl.

[5]) The interfacial tension coacervate/equilibrium liquid being very small, the drops are very easely deformed by convection currents and the rounding off (after these currents have come to a standstill) is a relatively slow process.

from which one or more segments are cut away. Cf. microphotograph *) *A* and fig. 1 *A* and *B*. Also smaller defects in the ideal spherical form may be observed (fig. 1 *C*).

2) It may happen that a number of drops drift along in a very conspicuous way: they are at short distances and have each (apart from the two outer ones) two plane faces which stand perpendicular to the direction in which the drops drift along (fig. 1 *D*). As the whole train of drops behaves as a mechanical unit the individual drops must be united by invisible films attached to the circular edges of the plane or slightly curved faces.

3) It has been observed that two drops of the shape of Fig. 1 *A*, formed an analogous mechanical unit, now of two members only when they happened to come with their plane faces up to a short distance from another [6]) (fig. 1 $E \rightarrow F$).

This observation makes it clear that the shape of Fig. 1 *A* must be caused by a closed film protruding in the equilibrium liquid and attached to the circular edge of the plane face as depicted in Fig. 1 *H*.

The film in question is invisible as a rule but with special efforts it was vaguely seen a few times and had indeed the shape of Fig. 1 *H* (dotted).

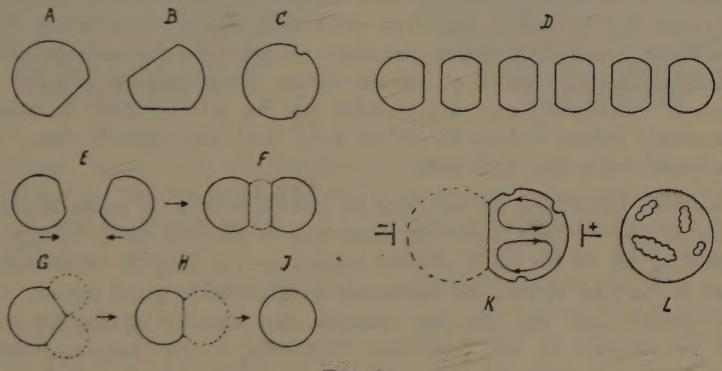

Fig. 1.

4) It may happen that a coacervate drop of the type of Fig. 1 *B* has its two plane faces situated in such a way that they seem to touch at one point. It then often occurs that the drop suddenly changes in shape into a drop of the type of Fig. 1 *A*. The two films of adjacent vesicles have then made real contact and have fused to a single one (see Fig. 1 $G \rightarrow H$). This sideway fusion is in principle the same phenomenon as occurs in the formation of drop chains (see above 2 and 3).

5) The presence of these films follows further from the observation that a mechanical disturbance outside the coacervate drop at a small distance from the plane face may result in a rounding-off of the coacervate drop (Fig. 1 $H \rightarrow I$). The protruding film which was not visible, was evidently destroyed then.

It is important that as a consequence of this destruction no other particulars can be seen at the place where the protruding vesicle was attached. It means that the liquid enclosed in this vesicle is wholly miscible with the surrounding equilibrium liquid. So the film in question has the property to be stable notwithstanding the fact that it divides two wholly miscible (and as we shall later see: identical) liquids.

6) Apart from electrophoresis (the direction of which indicates that the drops are

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*) The microphotographs have been taken by Mr W. W. H. WEYZEN, to whom we express our sincere thanks.

[6]) If drops having the shape of Fig. 1 *A* drift up against the glasswall and so come close to one another they may also unite into chains. In this case often networks of chains are formed, some drops having three faces functioning as branching points of the network.

negatively charged) we see in a d.c. field a number of phenomena inside the drop, some details of which are schematically given in Fig. 1 *K*. They are quite the same as those which are characteristic of negatively charged phosphatide coacervate drops, (which is a variant of the phenomena common to all negatively charged complex coacervate drops [7])).

Without entering here upon a discussion why small vacuoles are formed and why streaming inside the drops arises, we discuss here only the fate of those small vacuoles which happen to come into contact with the inside of the drop surface.

These vacuoles are unable to make their exit, but stick in the surface of the drop as protruding vesicles. The vesicle wall, however, is once more very thin and, as rule, not visible. Therefore, what is actually observed is only an indentation in the drop boundary, wholly comparable to Fig. 1 *C*. These indentations are transported by the moving surface of the drop (drops may creep in a d.c. field on a glass surface) towards one side (cathode side) of the drop and gradually form there by sideway fusing (see 4) the well known picture of Fig. 1 *A*, with only this difference that all coacervate drops now have one large plane face perpendicular to the electrical field on the same side of the drops apart from a number of small indentations (on their way of being transported) on other places. The same occurs if a d.c. field is applied on coacervate drops of the type Fig. 1 *A* and *B*. The plane faces pointing to chance directions all move to the cathode side of the coacervate drops, but now the large attaced vesicles are transported and not the small indentations. It therefore becomes very probable that the peculiar forms described, which are spontaneously present without the aid of a d.c. field, all originated from "vacuoles" originally imprisoned in the coacervate.

7) In harmony herewith is the fact, that by carefull addition of isoamylalcohol to the KCl containing oleate system coacervate drops may be obtained which contain vacuoles.

The rounding off of the finely divided coacervate (as long as convection currents are present in the total system, the coacervate is deformed to liquid threads, c.f. notes) is a slow process here and one may observe that actually equilibrium liquid gets entrapped as vacuoles in the coacervate. Evidently in the neighbourhood of the coacervation limit the entrapped vacuoles have not much chance in this case to come into contact with the inner surface of the drop-wall. Yet some have succeeded, the drops also showing plane faces here and there.

If another one or two drops of isoamylalcohol are added the coacervate drops do no longer contain vacuoles, and have now the typical shapes of fig. 1 *A* and 1 *B*. The factor which slowed down the meeting of vacuoles and inner surface of the drop, has evidently been removed.

8) Above (7) we have referred to vacuoles present in the coacervate drops obtained in the neighbourhood of the coacervation limit, but we did not yet describe the peculiar morphology in question. They are united in groups which because of their shape might be called *"articulated vacuoles"*. In reality these units are chains of vacuoles, in which the individual members are separated by invisible films (or by films that are visible only with great difficulty) at the positions of the articulations, fig. 1 *L*. Here once more an extremely thin film separates two miscible, nay identical liquids (the vacuole contents being equilibrium liquid).

The articulated vacuoles are most characteristic if we have obtained the P-coacervation with the smallest possible number of added drops of isoamylalcohol thus having just crossed the coacervation limit. The volume of the coacervate in this case being large in proportion to the volume of the equilibrium liquid, the latter is present in the form of articulated vacuoles embedded in a continuous layer of coacervate. The shape of the articulated vacuoles is still more characteristic if benzylalcohol is used in stead of isoamylalcohol. Cf. microphotograph *B*.

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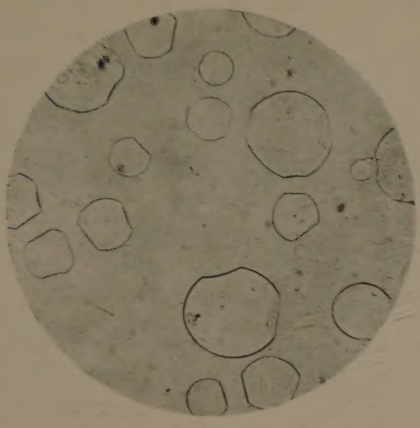
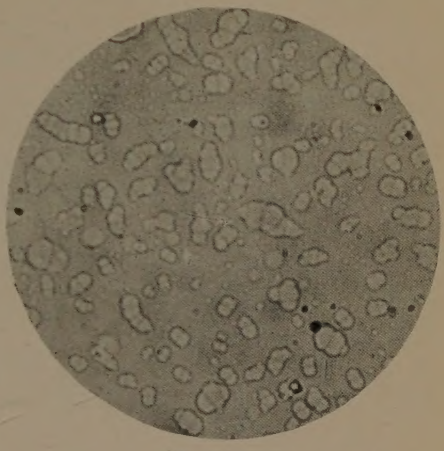
[7]) See H. R. KRUYT's Colloid Science II, loc. cit., p. 345, 452 and 479.

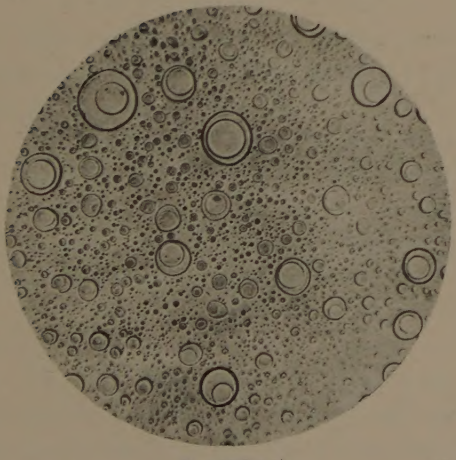
H. G. BUNGENBERG DE JONG and L. J. DE HEER: *Soap coacervates with special properties, hitherto only known in coacervates of phosphatides. I Oleate coacervates at a pH above* 12.



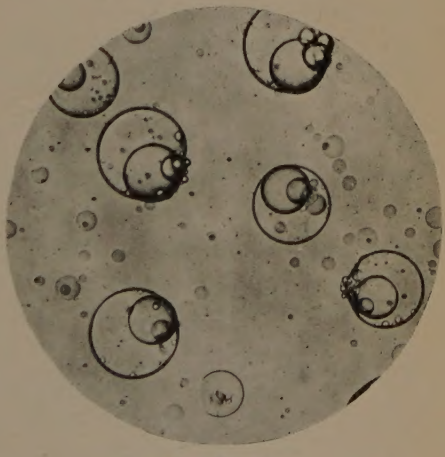
A (35 ×)
P-coacervate drops with superficially attached vesicles. The protruding vesicle wall is not visible.
(KCl containing oleate + isoamylalcohol)



B (106 ×)
Articulated vacuoles embedded in a continuous coacervate layer. The dark lines at the articulations are only special light effects, as the septa separating the individual members in a chain of vacuoles are not or hardly visible.
(KCl containing oleate + benzylalcohol)



C (105 ×)
Double drops, consisting of a coacervate shell and a central drop of the isoamylalcohol rich phase.
(KCl containing oleate + isoamylalcohol)



D (60 ×)
Triple drops, consisting of two coacervate shells and a central drop of the isoamylalcohol rich phase.
(KCl containing oleate + benzylalcohol after warming to 36°)

All the above very highly remarkable morphological particularities of these oleate coacervates, were observed on phosphatide coacervates many years ago already and we think that for the explanation of the nature of the peculiar films which play a prominent part in them, essentially the same line of reasoning can be followed as BUNGENBERG DE JONG and BONNER [8]) gave in the case of phosphatide coacervates.

We must start then from the contrast between ready fusibility of coacervate drops with each other on contact and the formation of stable films on contact of vacuoles with each other (above under 8) or of a vacuole with the inside of the drop surface. From this it follows already that the boundary coacervate/equilibrium liquid has different properties on the two sides. If, indeed, two such boundaries come into contact with one another while their surfaces are directed towards the equilibrium liquid, coalescence takes place (two coacervate drops meeting). On the other hand a stable film is formed when these boundaries encounter one another with their surfaces directed towards the coacervate side (two vacuoles meeting in the interior of the coacervate; vacuole and the inside of the surface of the drop meeting).

If we assume that there is a film of orientated oleate molecules at the coacervate/equilibrium liquid boundary, then it is understandable without any further argument that the two sides of this film bear very different characters. This renders it probable that the very peculiar films which may separate two indentical liquids are bimolecular. In section 5 we will return to the question with which side the monolayers lay together in this bimolecular film.

3. *Multiple coacervate drops.*

The analogy between phosphatide coacervates and P-coacervates of oleates goes still further, as also in the latter case formation of "multiple" drops may be observed.

Double drops are formed from the P-coacervates for instance when we gradually add more isoamylalcohol to the coacervated system (see microphotograph *D*). If still more is added, the outer shell, consisting of the P-coacervate diminishes in amount and finally disappears entirely, thus leaving the inner drops alone (fig. 2 *A*, $a \rightarrow b \rightarrow c \rightarrow d$).

The latter are rich in isoamylalcohol and dissolved oleate, but show no peculiar phenomena at all and cannot be regarded as coacervate drops.

The case described is a simple one: the formation of the double drops is evidently connected with a transgression of the solubility of isoamylalcohol in the total oleate system (being composed of: solubility in the $KCl + H_2O$ medium and the solubilizing capacity of the oleate, cf. section 5). The inner drops, however, are not indifferent, but absorb a certain amount of oleate,

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[8]) H. G. BUNGENBERG DE JONG and J. BONNER, Protoplasma **24**, 198 (1935) and a shorter summary in Proc. Kon. Akad. v. Wetensch., Amsterdam, **38**, 797 (1935).

and when more isoamylalcohol is added the oleate is displaced almost completely from the sphere of the aqueous medium (P-coacervate and equilibrium liquid) towards that of the medium rich in isoamylalcohol.

In harmony with this explanation is, that having arrived at the state *d* of fig. 2*A*, we may transgress the whole series of morphological states in a

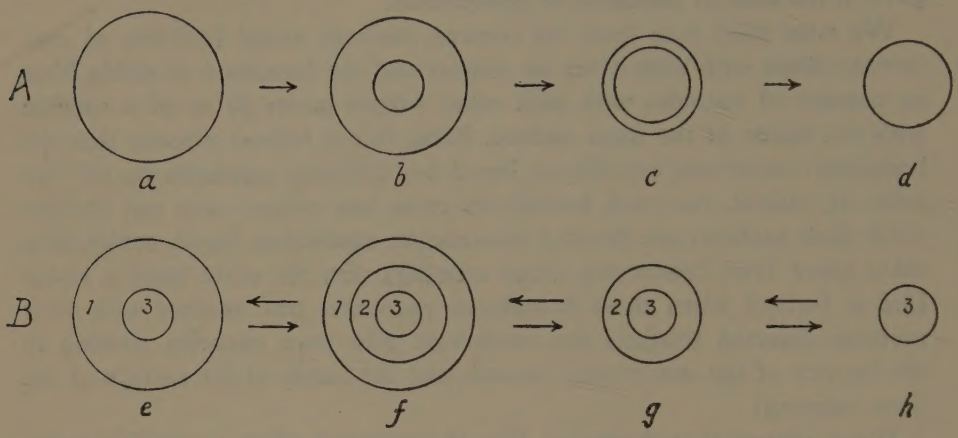

Fig. 2.

reverse direction (so $d \rightarrow c \rightarrow b \rightarrow a$) by now gradually adding 1 N. KCl solution, which is aproximately the electrolyte-water medium. Double drops may also be obtained by heating a system containing homogeneous P-coacervate drops ($a \rightarrow b$) while by cooling down the initial state is restored again ($b \rightarrow a$).

This may in principle be explained in the same way, as the solubility of isoamylalcohol in water is reduced on increasing the temperature.

A more complicated case is met with, if one uses benzylalcohol instead of isoamylalcohol to produce P-coacervation of the oleate. At room temperature additional benzylalcohol produces once more double drops. If we heat, however, this system under the microscope (e.g. with an electrical hair dryer, or more conveniently by applying an a.c. field [9])) a new liquid shell is produced at the boundary of the inner drops [10]). The triple drops thus formed (fig. 2*B*, $e \rightarrow f$) are composed of a central drop (3) mainly consisting of benzylalcohol, and two shells (1) and (2) (see microphotograph *D*) which both may be considered as oleate coacervates. At further increase of the temperature the outer shell (1) first disappears, and finally the shell left (2) and the drop rich in benzylalcohol (3) remains ($f \rightarrow g \rightarrow h$).

By subsequent cooling we return to the original state of double drops in a reverse direction ($h \rightarrow g \rightarrow f \rightarrow e$).

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[9]) Apart from the heat it produces the a.c. field, has no influence on the morphology of the coacervate drops.

[10]) Details as to the consecutive morphological changes (comprising also vacuolation processes) actually taking place when going from $E \rightarrow F$ in fig. 2, are omitted here. We hope to return to this interesting case of dynamic colloid morphology in a later communication.

We will not try to explain this more complicated case of benzylalcohol in details, we have recorded it here merely to emphasize once more on the phosphatide like character of the oleate coacervates obtained by adding so much of an appropriate polar non electrolyte that we are in the neighbourhood of its solubility. For also in phosphatide coacervates we may obtain double and triple coacervate drops [11]) by transgression of the solubility.

4. *Location of the regions in a KCl-isoamylalcohol diagram in which O- and P-coacervates are present.*

We shall have to judge microscopically the nature of the oleate system in a large number of combinations of KCl and isoamylalcohol concentrations. This can be simplified by using a method, the essential point of which consists in the microscopical inspection of the oleate system contained in a closed vessel and observing this through the glasswall. For this purpose we use a bent pyrex glasstube, containing the oleate system and being closed by a rubber stopper, which tube can be placed upon the stage of an inverted microscope as indicated in fig. 3.

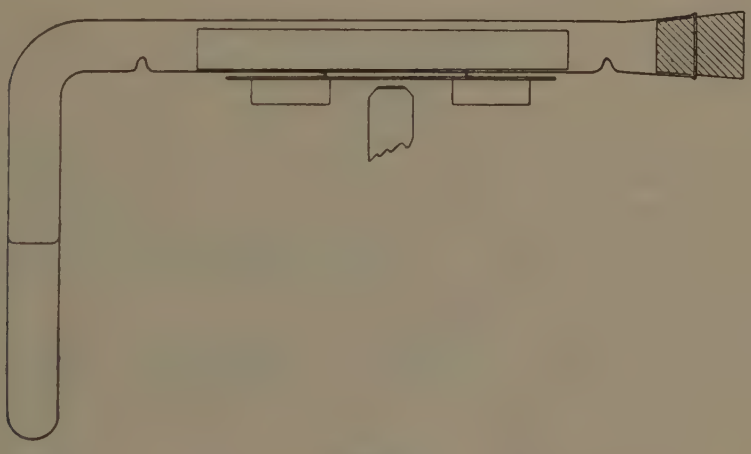

Fig. 3.

If an objective of small magnifying power is used and the horizontal limb lies in drop of water on a large cover-glass or on thin sheet of mica, one can obtain already a reasonable microscopic image of the oleate system close to the wall of the tube.

It is still inconvenient that the layer of the oleate system may be too thick, so that the image is too dark; while this layer may also continue to move for a very long time, which makes microscopic inspection either impossible or very difficult. Both inconveniences are satisfactorily removed by the presence of a cylindrical tube, open at both ends, in the horizontal limb. The oleate system in the capillary space between the bent tube and the smaller one inside the former can very satisfactorily be observed. Indeed, all morphological details recorded in the section 2 and 3 (apart from those in a d.c. field) can and actually have been investigated by using this method.

11) Reviewed in KRUYT's Colloid Science II, loc. cit, Chapter XI, § 6.

The possibility of examining microscopically the state of the oleate system in the closed tube has the advantage that for the objects of the present section we need not prepare separately a great many oleate systems containing different KCl and different isoamylalcohol concentrations. Instead of doing so we prepare an oleate KCl mixture in the bent tube to which we successively add small amounts, of isoamylalcohol and after rigorous shaking (causing the contents to run 50 or more times from one limb to the other) we can observe the state of the oleate system after each addition.

For the purpose of adding known amounts of isoamylalcohol we used the dripping pipette shown in fig. 4. The apparatus consists of three parts, a 1 cc micropipette *A*

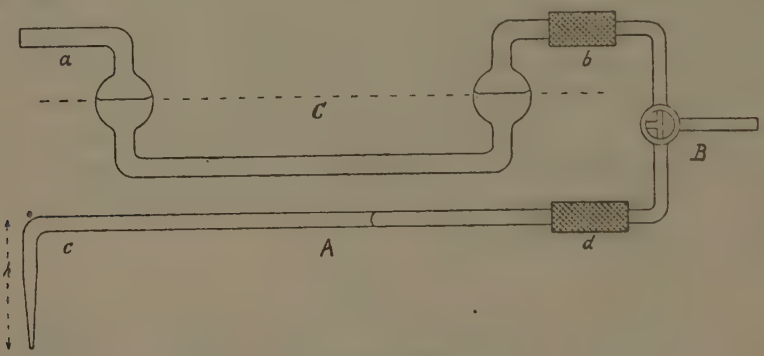

Fig. 4.

the lower part having been bent rectangularly, a centre part *B* with three way stopcock and an upper part *C* which serves as a flow resistance, and consists of two bulbs connected by a relatively wide capillary. The three parts are connected by short pieces of rubber vacuum tube, which gives flexibility at *b* and *d*. The apparatus is supported at *a*, *b*, *c* and *d* in such a way that the "horizontal" limb of *A* is actually in a horizontal position and that the levels of the very viscous liquid (paraffine oil or castor oil) are in one horizontal plane in the bulbs of *C*.

Drops of constant weight may be expected if the pipette (with stopcock turned as in fig. 4) is slowly dripping at a constant rate, This constant rate is obtained here by separating the two factors involved. The hydrostatic pressure *h* is independent of the degree of filling of micropipette *A* and the same practically also holds for the total viscous resistance (the resistance of the two liquids in series being practically that of the castor oil in the capillary connecting the two bulbs). By weighing 10 drops at different degrees of filling of the micropipette, we checked that the weight of the drops was practically constant (1 drop of isoamylalcohol = 12.4 mg). For filling the micropipette the three way stopcock is turned 90° anticlockwise and slight suction is applied at the horizontal tube of *B*. Then the stopcock is turned as in the figure and the first few drops which are delivered are disregarded. After use the stopcock is turned 90° clockwise, which allows the transported castor oil to flow back. This may be too slow a process and it may be necessary to readjust part *C* after some time (by changing the position of the support at a) in order to ensure that the levels in the bulbs remain in one horizontal plane.

Using the above experimental equipment we investigated at room-temperature ($\pm 16°$) the state of the oleate system (oleate concentration constant) at varying additions of isoamylalcohol for eight different KCl

concentrations. Starting from a stock solution of Na oleate, (4 g oleate + 90 ml H_2O + 10 ml 2 N. KOH) and a stock solution of 3.8 N. KCl, we introduce 5 ml oleate stock solution + a ml KCl 3.8 N. + (15—*a*) ml distilled water into the bent tube and investigate the nature of the system after adding ever more drops of isoamylalcohol. During the mixing after each addition care should be taken that the system is exposed to the warmth of the hands as less as possible. The results are recorded in Table I, which

TABEL I.

Number of drops of isoamylalcohol (1 drop = 12.4 mg) added to 20 ml 0.96 % oleate system containing KCl at which one of the successive states *a–h* (see text) is present at 16°.

C_{KCl}	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	log. C_{KCl}
0.19						61—68	70—78	80	0.28—1
0.28						57—62	63—69	70	0.45—1
0.47					46—47	48—54	55—59	60	0.67—1
0.56	0—16	(18—21)		(22—41)	42—43	46—50	53—55	57	0.76—1
0.76	0—11	12	13	16—37	39—40	42—48	49—52	53	0.88—1
1.14	0—2	3—4	5	6—31	32	34—39	40—		0.06
1.90	—	0—1		2—21	22	23—29	30—31	32	0.28
2.85	—	—	—	0—16	17	18—22	23—25	26	0.45

gives the number of drops at which a certain morphological state of the oleate system was observed.

These states, which we shall indicate, by *a*, *b*, *c* and so on are eight in number, two of which (*a* and *e*) are one phase systems, four (*b*, *d*, *f*, *h*) two phase systems and again two (*c* and *g*) three phase systems.

The kinds of phases in the successive states *a*—*h* are as follows:

a) *liquid*, representing a homogeneous solution of all components. This solution may often be very viscous and may exhibit marked elastic properties.

b) *liquid* + *liquid*, one of which contains practically all the oleate present and is therefore indicated "oleate coacervate", while the other liquid contains oleate only in very slight concentration and is therefore called "equilibrium liquid". In the terminology adopted in the preceding section the coacervate in question is the *"ordinary oleate coacervate"*.

c) *liquid* + *liquid* + *"semiliquid"*. The first two liquids are of the same kind as in *b* (equilibrium liquid + O-coacervate). The actual nature of the third phase cannot be identified with the above microscopic equipment. We know from former investigations however that this is a birefringent phase which has the nature of a fluid. Its viscosity must however be very high and we have therefore used the term "semiliquid". As a consequence of its high viscosity this phase appears as small granula, which often stick together to form small aggregates such as floccules, lumps, etc. The amount of this semiliquid phase increases at the cost of the O-coacervate by a further addition of isoamylalcohol.

d) *liquid* + *"semiliquid"*, these phases are elready present in *c* minus the O-coacervate. On further increase of the isoamylalcohol concentration the turbility of the total system may increase at first, but as a rule it decreases on further addition, as the amount of the "semiliquid" phase decreases.

e) *liquid*, the "semiliquid" phase, present in *c*) and *d*) now being dissolved.

f) *liquid* + *liquid*, being equilibrium liquid and an oleate coacervate, this time the *P-oleate coacervate*, the morphology of which has been investigated in section 2.

g) *liquid + liquid + liquid*, apart from the two liquids present in *f*) a third liquid, mainly consisting of isoamylalcohol, appears now. Drops of the P-coacervate take up drops of the latter phase to form the "double drops", which were studied in section 3. On further addition of isoamylalcohol the P-coacervate diminishes in volume.

h) *liquid + liquid*, one mainly consisting of a KCl solution in water the other which has practically taken up the oleate present in the system, consisting chiefly of isoamylalcohol.

In fig. 5 *A* these eight successive states of the oleate system have been pictured in a KCl-isoamylalcohol diagram by the regions *a*—*h*, the limiting curves of which can be drawn approximately from the data of Table I. For this purpose we took on every horizontal row the means of the last

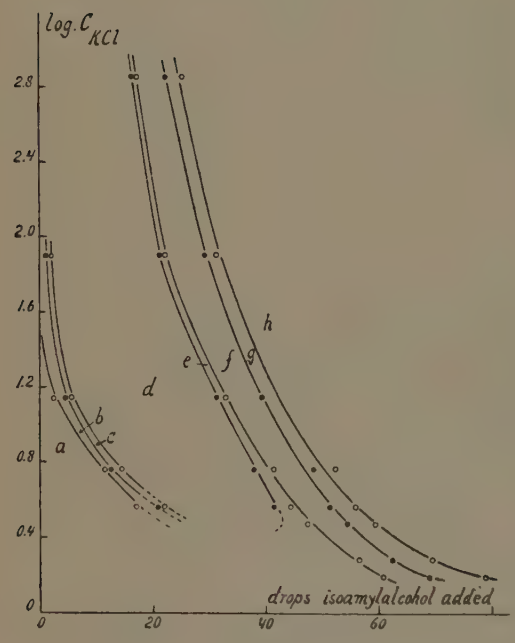

Fig. 5A.

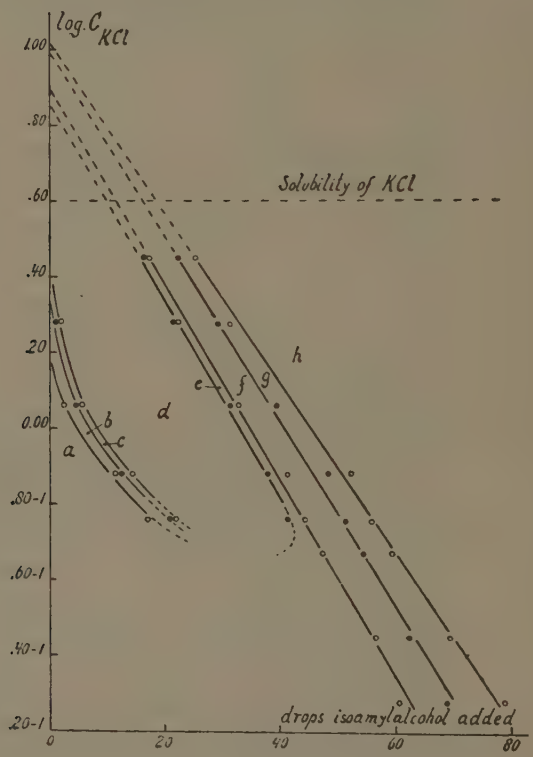

Fig. 5B.

figure in each column and the first in the next column to its right (thus e.g. the first row of the table gives as means 69 and 79).

The mean drop number values thus obtained have been plotted (alternatively as white and black dots) as abscissae against the corresponding KCl concentrations as ordinates. The position of the points is somewhat irregular so that simple curves cannot be drawn through all corresponding black and white points. Still we do not doubt that the limiting curves have the simple shape as given in fig. 5 *A*, the irregular position of the points being ascribed to the insufficient constancy of the room temperature.

We can read from this fig. 5 *A* how we can prepare O-coacervates

(present in the regions *b* and *c*) and P-coacervates (present in the regions *f* and *g*). First of all we must emphasize that in either case it is imperative that the *KCl concentration should not be too low*, for if isoamylalcohol is added to the oleate system in the absence of KCl, the states *b*, *c*, *d*, *f* and *g* cannot be realized. A considerable amount of amylalcohol can be dissolved in this case before we have reached its solubility, after which we suddenly enter region *h* (drops rich in isoamylalcohol appear).

We have not investigated how the regions *b*, *c*, *d*, *f* and *g* terminate downwards, but know for certain that those of *b*, *c*, and *d* disappear already below 0,5 N KCl, whereas the regions *f*, *g*, could still be obtained at 0.19 N KCl. It is therefore probable that in the lower part of the figure the regions *a* and *e* communicate.

We can further read from this figure that *O-coacervates* (*present in the regions b and c*) *can be realized already with KCl only*, the regions *a*, *b*, *c*, *d* touching the ordinate axis. *P-coacervates* (present in the regions *f* and *g*) *can only be obtained by the joint action of KCl and isoamylalcohol.* This conclusion however, seems not quite safe, as in our experiments the highest KCl concentration was 2.85 N, where as the solubility of KCl at room temperature amounts to approximately 4 N. If we re-draft fig. 5 *A*, using the logarithms of the KCl concentrations as ordinates, we get fig. 5 *B*, in which the curves limiting the regions *e*, *f* and *g* are practically straight lines now. Asuming for a moment that these straight lines might be extrapolated until they intersect the ordinate axis it would follow that P-coacervates could be obtained with KCl only. The minimum KCl concentration required (limiting line between *e* and *f*) would then amount to approximately log. $C_{KCl} = 0.90$, that is about 8 N. KCl. As at room temperature the solubility of KCl amounts to approximately 4 N only, (indicated in fig. 5 *B* by a dotted line at log. $C_{KCl} = 0.60$) these high concentrations cannot be realized.

That P-coacervation cannot really be reached with KCl only has been confirmed by adding isoamylalcohol to an oleate system which by the addition of solid KCl was saturated with KCl.

In the next section we will further investigate the rôle of isoamylalcohol in the production of P-coacervation.

(To be continued.)

Chemistry. — *Soap coacervates with special properties, hitherto only known in coacervates of phosphatides. I Oleate coacervates at a pH above* 12. II. By H. G. BUNGENBERG DE JONG and L. J. DE HEER.

(Communicated at the meeting of September 24, 1949.)

5. *Free and bound isoamylalcohol at the limit of the P-coacervation.*

In this section we intend to compare at a number of constant KCl concentrations the location of the boundary between the regions *e* and *f* (fig. 5) at three or more oleate concentrations. The experimental method followed in section 4, which would do for an approximate location of the boundaries in fig. 5, is not accurate enough for our present aims and is therefore replaced by a macroscopical method now, working in a thermostate at 20°. This method consists in measuring the relative volume of the coacervate at a number of isoamylalcohol additions very close to the above boundary between *e* and *f*. We shall describe this method more detailed with the help of the series of measurements at the constant KCl concentration of 0.475 N. We started from a stock oleate solution of the composition 29.95 g Sodium oleate of BAKER + 675 ml distilled water + 75 ml KOH 2 N. In the thermostate we placed three groups of glass-stoppered cylindrical measuring flasks of 50 ml capacity. In the first group we pipetted 15 ml oleate solution + 5 ml 1.9 N KCl, in the second 10 ml oleate solution + 5 ml H_2O + 5 ml 1.9 N KCl and in the third 5 ml oleate solution + 10 ml H_2O + 5 ml 1.9 N KCl.

In this way we obtained three series of mixtures in which the oleate concentration varies in the ratio of 3 : 2 : 1. We then added to each tube a different number of isoamylalcohol drops (with the apparatus of fig. 4) which lay in the neigbourhood of the coacervation limits of the three oleate concentrations at the given KCl concentration (roughly determined beforehand). After being mixed thoroughly the mixtures were left till the following day. The height of the column of coacervate was read then and expressed in per cent of the total volume in the tube (see Table II). If we plot these values as a function of the number of drops (fig. 6*A*) we can read in this graph the number of drops needed to reach the same coacervate volume (e.g. 60 %, 80 % and — by extrapolation — 100 %, the latter representing the coacervation limit). We now plot these numbers (also recorded in table II) as a function of the oleate concentration, cf. fig. 6*B*) in which the oleate concentrations are arbitrarely indicated by 1, 2 and 3).

The graph thus obtained shows that *the amount of isoamylalcohol needed to reach the same coacervate volume* (60 %, 80 % *or* 100 %) *is a linear function of the oleate concentration.*

This can be explained by assuming that the isoamylalcohol is present partly as free molecules dissolved in the KCl solution, and partly bound

in some way or other to the oleate (e.g. taken up in oleate "micelles"). The concentration of free isoamylalcohol can then be calculated from the

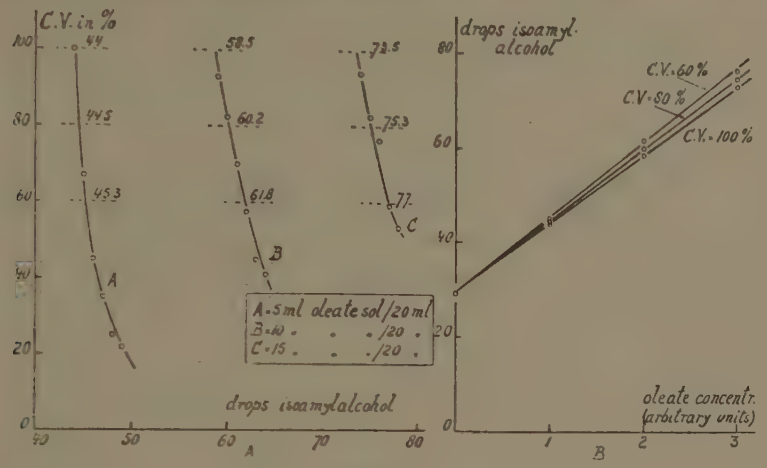

Fig. 6.

TABLE II.

Volume (in per cent) of the P-coacervate at three different oleate concentrations (KCl concentration = 0.475 N and temperature = 20°) as a function of the number of drops of isoamylalcohol added (1 drop = 12.4 mg).

15 ml oleate/20 ml		10 ml oleate/20 ml		5 ml oleate/20 ml	
drops isoamylalcohol per 20 ml system	coacervate volume per cent	drops isoamylalcohol per 20 ml system	coacervate volume per cent	drops isoamylalcohol per 20 ml system	coacervate volume per cent
69	(100)	59	93.0	44	(100)
74	94.0	60	82.5	45	67.0
75	82.5	61	70.0	46	45.0
76	76.5	62	57.5	47	35.0
77	59	63	45.0	48	25.0
78	53.5	64	41.5	49	23.5
from the above figures follow graphically:					
77	60	61.8	60	45.3	60
75.3	80	60.2	80	44.5	80
73.5	100 (coacervation limit)	58.5	100 (coacervation limit)	44	100 (coacervation limit)

distance between the origin and the intersection point of the straight line with the ordinate axis (29 drops per 20 ml; weight of 10 drops = 0.124 g; concentration therefore = 1.80 g per 100 ml). The slope of the straight line, on the other hand, makes it possible to calculate the ratio bound isoamylalcohol/oleate. For the coacervation limit (coacervate volume

exactly 100 %) 73.5 drops/20 ml are needed for the mixture containing 15 ml oleate solution, and 44 drops/20 ml for the mixture containing 5 ml oleate solution. The oleate in 10 ml stock solution (0.383 g = 1.262 millimol) binds therefore 73.5—44 = 29.5 drops = 4.152 millimol isoamylalcohol. This gives at the coacervation limit the ratio 4.152/1.262 = 3.29. Similarly we find at the coacervate volumes 80 % and 60 % the ratios 3.44 and 3.56.

As we have described the series at 0.475 N KCl more in details, it may suffice to give the results at other KCl concentration in a concise form. It has been invariably found that the quantity of amylalcohol just needed to start P-coacervation is a linear function of the oleate concentration. Cf. fig. 7, which is analogous to fig. 6 *B*, but in which we have only given the

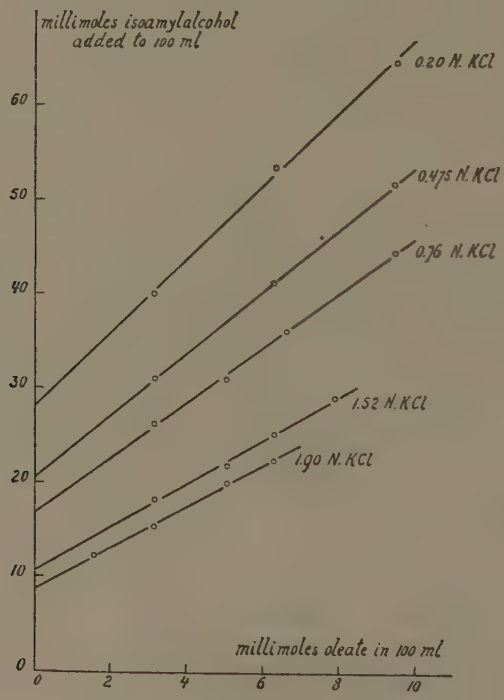

Fig. 7.

results for the coacervation limit (coacervate volume exactly 100 %) and in which the quantity of isoamylalcohol added is expressed in millimol per 100 ml; similarly the oleate concentration in millimol per 100 ml. The best fitting linear regression lines have been calculated by adopting a statistical method, from which we obtain for the concentration of free isoamylalcohol and the ratios of bound isoamylalcohol/oleate, the values recorded in the columns 2, 3 and 4 of the next survey.

In fig. 8 *A* we have plotted the figures of column 3 as a function of the KCl concentration (white dots). On the ordinate axis we have further indicated, with a black dot, the solubility (2.90) of isoamylalcohol in water

at 20°, expressed in g added to 100 ml water [12]). If we connect the black point on the ordinate axis with the first white dot (at 0.2 N KCl) a line is

KCl conc. N.	Conc. free isoamylalc.		bound isoamylalcohol / oleate (in molecular ratio)
	g added to 100 ml	millimol added to 100 ml.	
0.20	2.48	28.2	3.86
0.475	1.80	20.6	3.29
0.76	1.47	16.8	2.90
1.52	0.94	10.7	2.31
1.90	0.77	8.7	2.19

obtained (dotted) which practically coincides with what would appear as the probable course of the curve through the white dots if this curve is extrapolated to a KCl concentration zero. This suggests that the curve of fig. 8 *A* represents the solubility of isoamylalcohol at 20° as a function of the KCl concentration (in the absence of oleate) [13]).

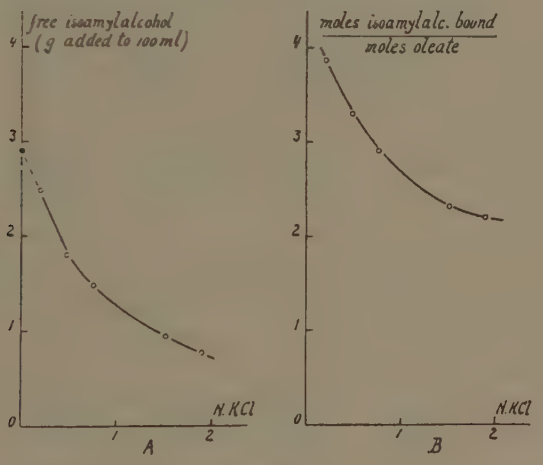

Fig. 8.

This then would mean consequently that P-coacervation sets in when the free isoamylalcohol standing in equilibrium with the amount of isoamylalcohol bound to the oleate, has reached its maximum value in the medium bathing the soap "micelles".

Now turning to the bound isoamylalcohol it self, cf. fig. 8 *B*, we observe

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[12]) Calculated from the value 2.82 g per 100 g solution at 20°, quoted in Seidell, Solubilities of inorganic and organic compounds, 2e Edition, New York 1928, see p. 1053.

[13]) Unfortunately no data are known to us in the literature concerning this solubility of isoamylalcohol in KCl solutions at 20°, to adstruck this assumption. Not in contradiction with it is the fact that we have actually observed that KCl diminished this solubility considerably.

that it is appreciable, and further that its molecular ratio depends on the KCl concentration. Yet we must lay stress on the fact that the P-coacervate is not at all a phase rich in isoamylalcohol, but one very rich in water.

When we consider e.g. a 1 % oleate system in which a P-coacervate has been obtained with a volume of 50 % or 25 % of the total system, its oleate contents cannot be larger than 2 % or 4 %. When we take the highest ratio of the survey (3.86) these coacervates would contain approximately 2.2 % or 4.4 % isoamylalcohol bound to the oleate only. The rest of the coacervate (93.8 or 91.6 %) must necessarily be constituted of the components of the equilibrium liquid (KCl solution saturated with isoamylalcohol). *The coacervate therefore mainly consists of a solution of KCl in water.*

This turns up a question, which we did not yet consider in section 2, viz. how the monolayer of oleate molecules present at the boundary P-coacervate/equilibrium liquid is oriented. We cannot yet give a definite answer to this problem, but we will warn for the too rash conclusion that these oriented oleate molecules "of course" have their polar heads directed towards the equilibrium liquid and their hydrocarbon chains towards the coacervate. For as we saw above there is not a very great difference at all in the constitutions of coacervate and equilibrium liquid (both being constituted mainly of a KCl solution in water). Therefore we may not consider the oleate coacervate as a much less polar liquid than the equilibrium liquid. It might even be considered as more polar than the other, taking into consideration that the concentration of the very polar oleate ions is many times higher in the coacervate than in the equilibrium liquid. This then would lead to the reverse position of the oleate molecules in the monolayer and as a consequence in the bimolecular membranes [14]). Further experiments, however, are needed to settle this problem.

Another question upon which the experiments of this section may shed some — though not a sufficient — light, is the rôle of isoamylalcohol in this kind of coacervation. As we have found that an appreciable amount of it is bound to the oleate at the appaerance of the P-coacervate, it seems likely that the inner structure of the soap micelles is affected by the isoamylalcohol molecules taken up.

Assuming here with BOOIJ [15]) that the micelles present in the O-coacervate are plates consisting of a double film of oriented oleate ions, the said affection might partially consist in a cleavage of the plates. In this way associations of oleate ions would be formed which have different properties at both sides and therefore could assemble at the interface P-coacervate/equilibrium liquid to form the postulated monomolecular layer.

The destruction of the micelles once begun may proceed still farther by

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[14]) Compare the discussion of the analogous question in the case of phosphatide coacervates in KRUYT's Colloid Science, Vol. II, Ch. XI, small print on p. 482.

[15]) H. L. BOOIJ in KRUYT's Colloid Science II, loc. cit. See Chapter XIV, § 5.

further addition of isoamylalcohol for instance into wholly free oleate ions. The fact that all morphological pecularities become less pronounced if after just passing the coacervation limit, more isoamylalcohol is added, gives support to the above idea, that it are really intermediate cleavage products of the plate micelles and not wholly free oleate ions, which play the prominent part in the described phenomena.

6. *Other examples of P-coacervation of soaps.*

The above experiments all refer to oleate at a pH $>$ 12 and KCl as salt. P-coacervation of oleate may be obtained at conditions which do no longer allow for O-coacervation: 1) at much lower pH (e.g. at pH 7.3, using a phosphate buffer), 2) when using LiCl or NaCl instead of KCl at room-temperature.

A qualitative investigation has further been made (still using isoamylalcohol and as a rule KCl) to see if P-coacervation is possible with other soaps. We could realize this with myristate, palmitate and stearate, further with "T-pol", a technical mixture of sulfate soaps (in this case $MgCl_2$ was used as salt) and with cetyltrimethylammouniumbromide (technical preparation). Thus the carbon chain may be altered, the COO' group may be replaced by another of the same electrical sign or by one of the reverse sign, without loss of the possibility of P-coacervation.

As phosphatides may be considered as amphoionic "soaps", we may generalize by saying that P-coacervation is possible with the three types of soaps: anionic, cationic and amphoionic.

7. *Summary.*

1) There are two kinds of oleate coacervates:

a) Coacervates showing a morphological behaviour hitherto known in phosphatide coacervates only and therefore called here *P-coacervates* (P stands for "phosphatide like").

b) Coacervates lacking this morphological behaviour and therefore called here O-coacervates (O stands for "ordinary").

2) The present communication dealing with P-coacervates of oleate obtained by the joint action of isoamylalcohol and KCl, first gives a description and microphotographs concerning the special morphology of the P-coacervate drops. The phenomena observed may be brought under three headings: *a*) abnormal behaviour of the vacuoles meeting one another or meeting the inner surface of the drop boundary, *b*) abnormal behaviour in a d.c. field, c) formation of multiple drops.

3) In the phenomena sub 2) under *a*) and *b*) the formation of very thin (invisible or visible only with extreme difficulty) membranes play the main rôle. They are only formed if two boundaries of coacervate/equilibrium liquid meet with their sides directed towards the coacervate. This

suggests the presence of a layer of oriented oleate molecules at the coacervate boundary, from which follows that the above mentioned thin membranes, having the property to separate two indentical liquids mainly consisting of water (+ KCl), must be bimolecular films.

4) When adding isoamylalcohol dilute oleate systems containing KCl may pass through eight successive states, each characterised by the number and kinds of the phases present. In two of these states the O-coacervate is present as one of the phases, in two others the P-coacervate. A KCl-isoamylalcohol diagram is given for 1 % oleate systems at approximately 16°, in which these eight states are represented as regions. The diagram shows that O-coacervates, can alraedy by obtained with KCl only, that P-coacervates, however, can only be obtained by the joint action of KCl and isoamylalcohol.

5) The amount of isoamylalcohol just needed to start P-coacervation at a given KCl concentration is a linear function of the oleate concentration. From this we calculated the concentration of isoamylalcohol which is present free in solution and the molecular ratio bound isoamylalcohol/oleate.

6) The results obtained sub. 5) were used for a preliminary discussion of two problems: *a*) why the P-coacervate boundary is the seat of an oriented layer of oleate molecules, *b*) the direction of the molecules in this layer.

7) P-coacervation is possible with the three types of "soaps": anionic, cationic and amphoionic soaps.

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Mathematics. — *The second pearl of the theory of numbers.* By J. G. VAN DER CORPUT and J. H. B. KEMPERMAN. (Second communication.)

(Communicated at the meeting of September 24, 1949.)

§ 2. *Generalisations of the $A + B$-theorem.*

In theorem 4, treated in the first communication, we have introduced an abstract addition. If we take there for the abstract sum of two numbers the ordinary product of these two numbers, theorem 4 furnishes the AB-theorem; in fact this is obvious if both A and B contain 1, and otherwise we may repeat here the argument by which we have shown in § 1, immediately after the formulation of theorem 4, that the $A + B$-theorem is a special case of theorem 4.

We find another application of theorem 4, if we put the abstract sum of two numbers a and b equal to $a + b + \lambda a b$, where λ is a given positive number. In this manner we obtain:

If γ and δ denote given numbers $\geqq 0$, and g and λ given positive numbers, and if the finite sets A and B formed by numbers $\geqq 0$ satisfy the inequalities

$$A(h) + B(h) \geqq 1 + \gamma \log (\lambda h + 1) + \delta$$

for $h = g$ and for each positive number $h < g$ belonging to A or B, then

$$(A + B + \lambda AB)(h) \geqq \gamma \log (\lambda h + 1) + \delta$$

for each positive number $h \leqq g$.

This theorem is an application of theorem 4 except in the case where the number 0 does not belong to both sets A and B; in the latter case the given inequality, applied with h equal to the smallest positive number which either is equal to g or belongs to A or B, gives $\gamma = \delta = 0$, since for that value of h we have $A(h) + B(h) \leqq 1$.

We give another application of theorem 4 in which we consider sets formed by complex numbers and where $S[h]$ denotes the number of elements of S with real part $< h$.

Let g be a given positive number and let $\varphi(h)$ be for $h > 0$ a slowly increasing and monotonic non-decreasing function.

If the finite sets A and B formed by complex numbers with real part $\geqq 0$ contain both at least one purely imaginary element and satisfy the inequalities

$$A[h] + B[h] \geqq 1 + \varphi(h)$$

for each positive number $h \leqq g$, then we have for these h

$$(A + B)[h] \geqq \varphi(h).$$

We may suppose in the proof that both A and B contain the number 0 and that neither A nor B contains an element $-ip$, where $p > 0$; in fact

if not, it is sufficient to submit A and B to a suitable translation parallel to the imaginary axis. Let us introduce the set G of the complex numbers either with positive real part, either of the form ip, where $p \geqq 0$. This set is ordered by the convention $x_1 + iy_1 < x_2 + iy_2$ first if $x_1 < x_2$ and secondly if $x_1 = x_2$, $y_1 < y_2$. We define $\varphi(x+iy)$ on G by the convention

$$\begin{aligned} \varphi(x+iy) &= \varphi(x) \quad \text{for } x > 0, \\ &= 0 \qquad \text{for } x = 0. \end{aligned}$$

Since the elements of A and B with real part $\geqq g$ do not occur in our proposition, it is not necessary to consider them, so that we may apply theorem 4, where H denotes the set of elements $\leqq g$ of G. This establishes the proof.

Theorem 4 is a generalization of the $A + B$-theorem which is valid for abstract sets G on which a commutative and associative addition has been defined. We have chosen the formulation above of theorem 4, because this generalization of the $A + B$-theorem has the same proof as the $A + B$-theorem itself. Nevertheless we were conscious of the fact that the assumptions of theorem 4 may be replaced by much weaker conditions. We will do this in this communication with the disadvantage that the argument given in the first communication must be replaced by a much more subtle mode of proof. In order to avoid unnecessary complications we restrict us in what follows to the case where $\varphi(h)$ is monotonic non-decreasing on the ordered set G, on which it has been defined.

First we state that it is not necessary to define the sum $a + b$ for each pair of elements a and b belonging to G. It is sufficient to suppose first that, if a is an arbitrary element of A and b an arbitrary element of B, then G contains an uniquely defined element $a + b$. These elements form a subset $A + B$ of G. Moreover we suppose that the sum $g + b$ has been defined uniquely for each element g of $A + B$ and for each element b of B.

The double condition that the addition is commutative and associative may be replaced by the simple condition

$$(a + b) + b' = (a + b') + b$$

valid for each element a of A and each pair of elements b and b' of B.

In view of these facts we say that we have defined on a set G a *partial addition* with respect to the non-empty subsets A and B of G, if for every element b of B the sum $g + b$ denotes an uniquely defined element of G, first for each element g of A and secondly for each element g of $A + B$, such that the following four *addition properties* are valid:

1. If a is an element of A and b and b' are elements of B, then

$$(a + b) + b' = (a + b') + b.$$

2. If b is an element of B, and g and g' belong either both to A or both to $A + B$, then

$$g + b = g' + b \text{ implies } g = g'.$$

3. If a is an element of A, and b and b' are elements of B, then

$$a + b = a + b' \text{ implies } b = b'.$$

4. B contains an element b_0 such that it is impossible to find in B an element $b \neq b_0$ and in $A + B$ a finite number of elements

$$g_0, g_1, \ldots, g_{m-1}, g_m = g_0 \qquad (\text{where } m \geqq 1)$$

with the properties

$$g_\mu + b = g_{\mu+1} + b_0 \qquad (\mu = 0, 1, \ldots, m-1).$$

Using this convention we may generalise theorem 4, apart from the monotony of $\varphi(h)$, in the following manner:

Theorem 5. Condition 1. *Let be defined on an ordered set G a partial addition with respect to two given finite non-empty subsets A and B of G. We denote the smallest element of A and B respectively by a_0 and b_0, where b_0 is the particular element of B mentioned in the fourth addition property.*

Let be given on G a real monotonic non-decreasing function $\varphi(g)$ with

$$\varphi(g) = \varphi(g + b_0);\ \varphi(b) \leqq \varphi(g + b);\ \varphi(b) = \varphi(a_0 + b),$$

where g is an arbitrary element of A or $A + B$ and where b is an arbitrary element $\neq b_0$ of B.

Further we assume $\varphi(a_0) \leqq 0$ if $A + B'$ contains the element $a_0 + b_0$ and also if $A + B'$ contains at least one element $< a_0$; here B' is the set formed by the elements $\neq b_0$ of B.

Condition 2. *Finally we assume*

$$\varphi(g + b) \leqq \varphi(g) + \varphi(b),$$

where g is an arbitrary element of A or $A + B$ and where b is an arbitrary element $\neq b_0$ of B.

Assertion. *If the inequalities*

$$A(h) + B'(h) \geqq \varphi(h)$$

are valid for each element h belonging to a given subset H of G, which contains every element $> a_0$ of A and every element $> b_0$ of B, then we have for each element $h > a_0 + b_0$ of H

$$(A + B)(h) \geqq \varphi(h).$$

Remark. The inequality to be proved is obvious for the elements $h \leqq a_0$ of H, since they satisfy $\varphi(h) \leqq 0$, as we will show now.

If B' is empty, and also if $h \leqq b'$, where b' denotes the smallest element of B', we have $B'(h) = 0$, and moreover $A(h) = 0$ by $h \leqq a_0$, hence

$$0 = A(h) + B'(h) \geqq \varphi(h)$$

Let us now consider the elements h of H with $b' < h \leqq a_0$. In virtue of the monotony it is sufficient to prove $\varphi(a_0) \leqq 0$. This inequality follows from

condition 1 of theorem 5, if $A + B'$ contains an element $< a_0$, so that we may suppose $a_0 + b' \geqq a_0$, and in that case we obtain

$$\varphi(a_0) \leqq \varphi(a_0 + b') = \varphi(b') \leqq A(b') + B'(b') = 0.$$

If a_0 is an element of H, the inequality of theorem 5 to be proved is valid for each element h of H, since in that case we just have found $\varphi(a_0) \leqq 0$ and therefore $\varphi(a_0 + b_0) = \varphi(a_0) \leqq 0$, hence $\varphi(h) \leqq 0$ for each element $h \leqq a_0 + b_0$ of H.

Theorem 4 with the supplementary condition that $\varphi(g)$ is monotonic non-decreasing is a particular case of the proposition 5, since in that case the conditions of the latter theorem are satisfied. In fact, it is obvious that the properties 1, 2 and 3 mentioned in the definition of the partial addition are valid, and from the fact that $b > 0$ implies $g + b > g$, it follows that also property 4 is valid.

Till now we have restricted us to theorems without weights, but we may give to each element g of G a weight $f(g) \geqq 0$. In that case $S(h)$ does not denote the number of elements $s < h$ of S, but the sum of the weights $f(s)$ of these elements s; in formula

$$S(h) = \sum_{s<h} f(s). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

By putting $f(g) = 1$ the following theorem transforms into theorem 5.

Theorem 6. Condition 1. *Let us suppose that condition 1 of theorem 5 is fulfilled.*

Condition 2. *We give to each element g of G a weight $f(g) \geqq 0$, such that*

$$f(g) = f(g + b_0); \quad f(b) \leqq f(g + b),$$

where b is an arbitrary element $\neq b_0$ of B and where g is an arbitrary element of A or $A + B$.

Condition 3. *Further we assume*

$$\varphi(g + b) \leqq \varphi(g) + \varphi(b); \quad f(g + b) \geqq f(g)$$

where b is an arbitrary element $\neq b_0$ of B and where g is an arbitrary element of A or $A + B$.

The assertion is the same as in theorem 5, apart from the fact, that $A(h)$, $B'(h)$ and $(A + B)(h)$ are defined by (14).

In order to generalise this result it is recommendable to introduce not only one pair of functions $\varphi(g)$, $f(g)$, but a finite or infinite number of such pairs. In this manner we obtain the following theorem, which we consider as the principal proposition of these communications, the proof of which will be given in § 3.

Theorem 7. *Suppose that on a given ordered set G with two given non-empty finite subsets A and B a finite or infinite number of pairs of functions $\varphi(g)$ and $f(g)$ have been defined, such that the conditions 1 and 2 of theorem 6 are satisfied for each of these considered pairs.*

If $\varphi(g)$, $f(g)$ is an arbitrary considered pair of functions and if b denotes an arbitrary element $\neq b_0$ of B, then we suppose moreover that it is possible to find among the considered pairs of functions at least one pair, $\Phi(g)$, $F(g)$ (depending on b and on the choice of the pair $\varphi(g)$, $f(g)$) such that

$$\varphi(g+b) \leq \Phi(g) + \varphi(b); \quad f(g+b) \geq F(g)$$

for each element g of A or $A+B$.

If under these conditions each pair of functions $\varphi(g)$, $f(g)$ satisfies the inequality

$$A(h) + B(h) \geq \varphi(h)$$

for each element h of a given subset H of G which contains all elements $> a_0$ of A and all elements $> b_0$ of B, then we have for each considered pair of functions $\varphi(g)$, $f(g)$ and for each element $h > a_0 + b_0$ of H

$$(A+B)(h) \geq \varphi(h).$$

Of course $A(h)$ etc. is defined by (14). The remark following immediately after theorem 5 is here also true.

As an application we give:

Theorem 8. *Let G be an ordered set containing a smallest element denoted by 0, on which a commutative and associative addition has been defined, with $g+0=g$ and $g+g^* > g$ for $g^* > 0$, such that*

$$g+g' = g+g'' \text{ implies } g' = g''.$$

Let $\varphi^*(g)$ and $f^*(g) \geq 0$ be monotonic non-decreasing functions on G and let H be a subset of G.

If the finite subsets A and B of H, containing both the element 0, satisfy for each element g of G and each positive element h of H the inequality

$$\sum_{a<h} f^*(a+g) + \sum_{0<b<h} f^*(b+g) \geq \varphi^*(h+g) - \varphi^*(g), \quad . \quad . \quad (15)$$

then we have for these element h and g also

$$\sum_{a+b<h} f^*(a+b+g) \geq \varphi^*(h+g) - \varphi^*(g).$$

Proof. For each pair of elements g and g^* of G we put

$$\varphi(g) = \varphi^*(g+g^*) - \varphi^*(g^*) \text{ and } f(g) = f^*(g+g^*),$$

so that $\varphi(g)$ and $f(g)$ depend not only on g, but also on g^*. In order to prove that the conditions of theorem 7 are satisfied for these pairs $\varphi(g)$, $f(g)$, it is sufficient to observe that the pair formed by the functions

$$\Phi(g) = \varphi^*(g+(g^*+b)) - \varphi^*(g^*+b) \text{ and } F(g) = f^*(g+(g^*+b))$$

possess the required properties

$$\varphi(g+b) \leq \Phi(g) + \varphi(b) \text{ and } f(g+b) \geq F(g);$$

these relations are valid even with the sign of equality. Now the assertion of theorem 7 furnishes theorem 8.

Let us show that theorem 4 with the supplementary condition that $\varphi(g)$ is a monotonic non-decreasing function is a particular case of theorem 8. We remark that if the conditions of theorem 4 with the monotonic non-decreasing function $\varphi(g)$ are satisfied, they remain valid if $\varphi(g)$ is replaced by

$$\varphi^*(g) = \text{max. } (0, \varphi(g)).$$

In fact it is sufficient to prove that the monotonic function $\varphi^*(g)$ is slowly increasing on the set of positive elements of G, that is

$$\varphi^*(g_1 + g_2) \leqq \varphi^*(g_1) + \varphi^*(g_2)$$

for $g_1 > 0$ and $g_2 > 0$, and this inequality is obvious as we have either

$$\varphi^*(g_1 + g_2) = 0 \leqq \varphi^*(g_1) + \varphi^*(g_2)$$

or

$$\varphi^*(g_1 + g_2) = \varphi(g_1 + g_2) \leqq \varphi(g_1) + \varphi(g_2) \leqq \varphi^*(g_1) + \varphi^*(g_2).$$

From this result it follows, that the conditions of theorem 8 are satisfied with $f^*(g) = 1$, and with $\varphi^*(0) = 0$; further

$$A(h) + B'(h) \geqq \varphi^*(h) \geqq \varphi^*(h + g) - \varphi^*(g)$$

for every positive element h of H and for every element g of G. Therefore (15) is satisfied, so that the assertion of theorem 8, applied with $g = 0$, gives

$$(A + B)(h) \geqq \varphi^*(h) \geqq \varphi(h)$$

for every positive element h of H.

In theorem 8 we assume that the inequality (15) is valid for each element g of G and each positive element h of H. That in some cases it is not necessary to assume all these inequalities, appears from the following example. The special case, where $\gamma(h)$ is constant and where H is the set of non-negative integers below a given bound, has been proved by J. G. VAN DER CORPUT [8]).

Let G be a set of numbers $\geqq 0$, such that on G the addition is always possible, and let $f(g)$ be a positive monotonic non-decreasing function on G. Let $\gamma(h)$ be a monotonic non-increasing function $\geqq 0$ defined on a given finite subset H of G.

Further we suppose

$$f(h+g)\, f(h') \geqq f(h'+g)\, f(h) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

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[8]) On sets of integers, Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **50**, 345 (1947). Also: Indagationes Mathematica 9, 203 (1947). VAN DER CORPUT assumes $f^2(h+1) \geqq f(h)\, f(h+2)$ $(h = 1, 2, \ldots)$ in stead of (16). It is clear that his assumption implies (16) for $g = 1, 2, \ldots$; in fact $\frac{f(h+1)}{f(h)}$ and therefore also

$$\frac{f(h+g)}{f(h)} = \frac{f(h+g)}{f(h+g-1)} \cdots \frac{f(h+1)}{f(h)} \qquad (g = 1, 2, \ldots)$$

are monotonic non-increasing functions of h.

for every element g of G and every pair of elements h and h' of H, such that $h < h'$.

If both subsets A and B of H contain the number 0 and satisfy the inequality

$$\sum_{a<h} f(a) + \sum_{0<b<h} f(b) \geqq \sum_{h'<h} \gamma(h')\, f(h') \quad . \quad . \quad . \quad . \quad (17)$$

for each positive element h of H, then we have for these h

$$\sum_{a+b<h} f(a+b) \geqq \sum_{h'<h} \gamma(h')\, f(h') ; \quad . \quad . \quad . \quad . \quad . \quad (18)$$

the sum $\sum_{h'<h}$ is extended over all elements $h' < h$ of H.

Proof. It is sufficient to prove that the inequality (17), valid for each positive element h of H, implies

$$\sum_{a<h_1} f(a+g) + \sum_{0<b<h_1} f(b+g) \geqq \sum_{h'<h_1} \gamma(h'+g)\, f(h'+g) \quad . \quad . \quad (19)$$

for each positive element h_1 of H and each element g of G. In fact the right hand side of (19) is equal to $\varphi^*(h_1+g) - \varphi^*(g)$, where $\varphi^*(g)$ denotes the function $\sum_{h'<g} \gamma(h')\, f(h')$, which is monotonic non-decreasing on G; the conditions of theorem 8 are therefore fulfilled, so that the assertion of that theorem, applied with $g = 0$, furnishes the inequality (18) for every positive element h of H.

We put for each positive element $h \leq h_1$ of H

$$\varrho_h = \frac{f(h^*+g)}{f(h^*)} - \frac{f(h+g)}{f(h)} \quad \text{for } h < h_1$$

$$= \frac{f(h^*+g)}{f(h^*)} \quad \text{for } h = h_1,$$

where h^* denotes the greatest element $< h$ of the finite set H (such an element exists, since h is greater than the element 0 of H). As the positive function $f(h)$ satisfies the inequality (16), we have $\varrho_h \geqq 0$.

From

$$\sum_{a<h\leqq h_1} \varrho_h = \frac{f(a+g)}{f(a)}$$

valid for each element $a \geqq 0$ and $< h_1$ of A, we deduce

$$\sum_{h\leqq h_1} \varrho_h \sum_{a<h} f(a) = \sum_{a<h_1} f(a) \sum_{a<h\leqq h_1} \varrho_h = \sum_{a<h_1} f(a+g),$$

and similarly

$$\sum_{h\leqq h_1} \varrho_h \sum_{0<b<h} f(b) = \sum_{0<b<h_1} f(b+g).$$

Moreover

$$\sum_{h\leqq h_1} \varrho_h \sum_{h'<h} \gamma(h')\, f(h') = \sum_{h'<h_1} \gamma(h')\, f(h') \sum_{h'<h\leqq h_1} \varrho_h =$$

$$= \sum_{h'<h_1} \gamma(h')\, f(h'+g) \geqq \sum_{h'<h_1} \gamma(h'+g)\, f(h'+g).$$

Multiplying both members of (17) with $\varrho_h \geqq 0$ and adding the thus found inequalities, we obtain the inequality (19), which was to be proved.

Although the set G occurring in theorem 6 is supposed to be ordered, it is nevertheless possible to apply that proposition on sets for which no order has been defined, for instance:

Theorem 9. *Consider a commutative additive semigroup $\mathfrak{G}$ without any finite non-trivial sub-semigroup. Define $f(g) \geqq 0$ and $\varphi(g)$ on $\mathfrak{G}$ with $\varphi(0) = 0$ and*

$$\varphi(g) \leqq \varphi(g+g') \leqq \varphi(g) + \varphi(g'); \quad f(g) \leqq f(g+g'),$$

where g and g' denote arbitrary elements of $\mathfrak{G}$.

Let σ be positive. If the finite subsets A and B of $\mathfrak{G}$ contain both the element 0 and satisfy the inequalities

$$\sum_{\varphi(a)<\varrho} f(a) + \sum_{\substack{\varphi(b)<\varrho \\ b \neq 0}} f(b) \geqq \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

for each number $\varrho \leqq \sigma$, then we have for these numbers ϱ

$$\sum_{\varphi(a+b)<\varrho} f(a+b) \geqq \varrho.$$

R e m a r k. We call a set $\mathfrak{H}$, on which an associative addition has been defined, a semigroup, if it contains an element 0 with $h + 0 = 0 + h = h$, valid for each element h of $\mathfrak{H}$, and if further both $h + h' = h + h''$ and $h' + h = h'' + h$ imply $h' = h''$. The trivial sub-semigroup is formed by the element 0.

P r o o f. Without loss of generality we may suppose that $\varphi(g)$ is everywhere $\geqq 0$ and $\leqq \sigma$, for otherwise we introduce $\varphi^*(g)$ with

$$\begin{aligned} \varphi^*(g) &= 0 && \text{if} && \varphi(g) \leqq 0 \\ \varphi^*(g) &= \varphi(g) && \text{if} && 0 \leqq \varphi(g) \leqq \sigma \\ \varphi^*(g) &= \sigma && \text{if} && \varphi(g) \geqq \sigma, \end{aligned}$$

in stead of $\varphi(g)$, just as we have done in the proof that theorem 4 is a consequence of theorem 8.

We apply theorem 6. If $\mathfrak{G}$ contains at least one element g^* with $\varphi(g^*) = \sigma$, then we choose in this theorem $G = \mathfrak{G}$. Otherwise we denote by G the set, formed by the elements of the given semigroup $\mathfrak{G}$ and further a new element g^*; in this case we put $\varphi(g^*) = \sigma$ and we choose $f(g^*)$ arbitrarily $\geqq 0$.

On G we define a transitive order, such that 0 is the smallest element of G and that $\varphi(g)$ is monotonic non-decreasing on G.

First we show that we have on G a partial addition with respect to the given subsets A and B. For that purpose it is sufficient to examine the fourth addition property. Suppose therefore that the relations ($m \geqq 1$)

$$g_\mu + b = g_{\mu+1} \qquad (\mu = 0, 1, \ldots, m-1)$$

with $g_0 = g_m$ are valid, where $b \neq 0$ and $g_0, \ldots g_m$ denote elements of the commutative semigroup $\mathfrak{G}$. Then we have $m\,b = 0$, and this is impossible, since $\mathfrak{G}$ does not possess a finite non-trivial sub-semigroup.

The conditions of theorem 6 are satisfied with $H = G$. In order to apply the assertion of that theorem, we must deduce the inequality

$$\sum_{a<h} f(a) + \sum_{0<b<h} f(b) \geqq \varphi(h) \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

for each element h of G, and this is obvious as $\varphi(a) < \varphi(h)$ implies $a < h$ and (21) follows from (20), applied with $\varrho = \varphi(h) \leqq \sigma$.

The assertion of theorem 6 gives

$$\sum_{a+b<h} f(a+b) \geqq \varphi(h)$$

for each element h of G. To show that this result implies the inequality which is to be proved, we introduce the smallest element h^* with $\varphi(h^*) \geqq \varrho$, which is equal to g^* or belongs to $A + B$; such an element exists by $\varphi(g^*) = \sigma \geqq \varrho$. From this definition of h^* it follows that each element $a + b < h^*$ of $A + B$ satisfies the inequality $\varphi(a + b) < \varrho$, hence

$$\sum_{\varphi(a+b)<\varrho} f(a+b) \geqq \sum_{a+b<h^*} f(a+b) \geqq \varphi(h^*) \geqq \varrho.$$

Zoology. — *Rhizocephalan parasites of crabs of the genus Metopograpsus.* By H. BOSCHMA.

(Communicated at the meeting of September 24, 1949.)

The present paper contains notes on Rhizocephala of the crabs *Metopograpsus messor* (Forsk.), *M. oceanicus* Jacq. & Luc. and *M. quadridentatus* Stimps. The three species have one common parasite, *Sacculina yatsui* Boschma, but, moreover, two of the crabs may be infested by another species of parasite. In *M. messor* this other parasite is *Sacculina plana* Boschma, in *M. quadridentatus* it is *Sacculina imberbis* Shiino. Whilst in *Sacculina yatsui* the external cuticle of the mantle possesses comparatively large excrescences the cuticle of the two other species of *Sacculina* is devoid of excrescences.

In the two species of *Metopograpsus*, therefore, an infestation with parasites is found similar to that in *Pachygrapsus crassipes* Randall, parasitized by *Sacculina yatsui* and *S. confragosa* (cf. BOSCHMA, 1935, 1936). But here the case is still more complicated, as shown by SHIINO (1943), who described *Sacculina imberbis* as a third species of Rhizocephalan parasite of *Pachygrapsus crassipes*. Matters now are becoming rather complicated for identification, as *S. imberbis* as well as *S. confragosa* do not possess excrescences, so that their differences are to be found in sectioned material only.

Up till now no Rhizocephalan parasites of crabs of the genus *Metopograpsus* have been commented upon. Though these parasites belong to already known species some of their peculiarities may be dealt with here to draw attention to individual variation and to show differences from previously described material.

Sacculina yatsui Boschma

Snellius Expedition, Kambang near Timor, November 28, 1929, 1 ex. on *Metopograpsus messor* (Forsk.), dimensions of the parasite $12\frac{1}{2} \times 6\frac{1}{2} \times 2$ mm.

Snellius Expedition, Kaledupa, Tukang Besi Islands, August 27, 1930, 1 ex. on *Metopograpsus messor* (Forsk.), dimensions of the parasite $9 \times 4\frac{1}{2} \times 2$ mm.

Snellius Expedition, Tanah Djampea, February 21, 1930, 1 ex. on *Metopograpsus oceanicus* Jacq. & Luc., dimensions of the parasite $11\frac{1}{2} \times 5\frac{1}{2} \times 2$ mm.

Dodinga, Halmahera, VON MARTENS leg. (collection Zoological Museum Berlin), 1 ex. on *Metopograpsus quadridentatus* Stimps., dimensions of the parasite $17 \times 10 \times 5$ mm.

Japan, HILGENDORF leg. (collection Zoological Museum Berlin), 1 ex. on *Metopograpsus quadridentatus* Stimps., dimensions of the parasite $15\frac{1}{2} \times 8\frac{1}{2} \times 5\frac{1}{2}$ mm.

The shape of the specimens is rather variable. The two specimens on *Metopograpsus messor* (fig. 1 *a*, *b*) are flattened and broadly oval, their mantle opening does not noticeably project above its surroundings, the

mantle shows a few grooves chiefly in its marginal part. The specimen on *M. oceanicus* (fig. 1 *c*) too is flattened, it is more or less panduriform, whilst the mantle opening lies at the top of a short tube; here again the mantle shows a few grooves. The specimens on *M. quadridentatus* are considerably thicker;

Fig. 1. *Sacculina yatsui*, left side of five specimens. *a*, on *Metopograpsus messor* from Kambang; *b*, on *Metopograpsus messor* from Kaledupa; *c*, on *Metopograpsus oceanicus*; *d*, on *Metopograpsus quadridentatus* from Dodinga; *e*, on *Metopograpsus quadridentatus* from Japan. *a*, *c*, × 4; *b*, × 6; *d*, *e*, × 3.

in the Halmahera specimen (fig. 1 *d*) the anterior part is much broader than the posterior region, in the specimen from Japan (fig. 1 *e*) the posterior region is slightly broader than the anterior part. In these two specimens the surroundings of the mantle opening are raised above the mantle. In general the shape of these parasites is not strikingly different from that of specimens of *Sacculina yatsui* on *Pachygrapsus crassipes* (cf. BOSCHMA, 1936, fig. 1; SHIINO, 1943, fig. 1 F).

The male organs of the various specimens show some individual variation.

In the specimen on *Metopograpsus oceanicus* the ventral parts of the vasa deferentia are wide (fig. 2 *a*), on their inner wall they have a few ridges only. Towards the dorsal region they gradually become narrower and pass into the testes. Fig. 2 *b* shows the region in which the right vas deferens penetrates into the testis of this side. The right testis is slightly larger than the left.

In the specimen on *Metopograpsus messor* from Kaledupa the vasa

deferentia have a fairly wide lumen, their inner wall has numerous ridges (fig. 2 *c*). The left testis reaches its largest size in the region in which the right vas deferens passes into its testis (fig. 2 *d*). Towards the dorsal region the right testis becomes much larger (fig. 2 *e*).

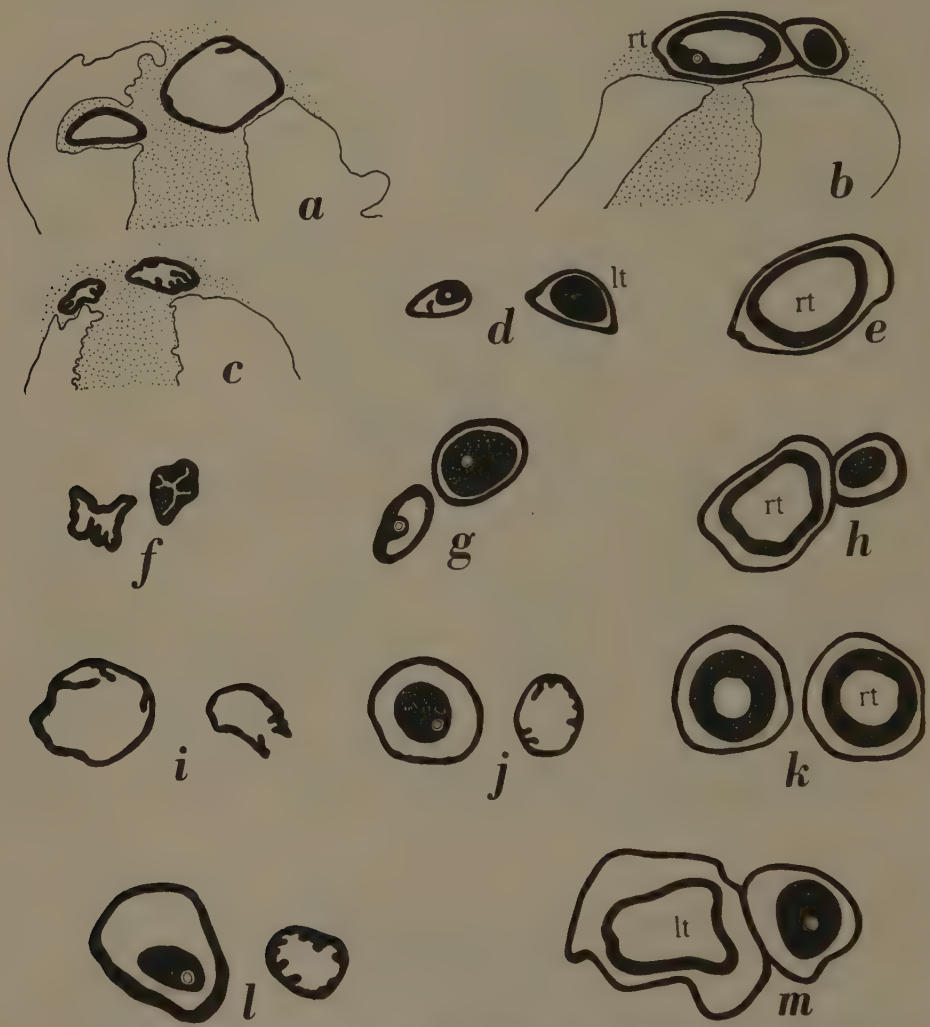

Fig. 2. *Sacculina yatsui*, transverse sections of the male genital organs of five specimens. *a, b,* on *Metopograpsus oceanicus*; *c—e,* on *Metopograpsus messor* from Kaledupa; *f—h,* on *Metopograpsus messor* from Kambang; *i—k,* on *Metopograpsus quadridentatus* from Dodinga, *l, m,* on *Metopograpsus quadridentatus* from Japan. *lt,* left testis; *rt,* right testis. × 30.

The vasa deferentia of the specimen on *Metopograpsus messor* from Kambang are very narrow as their inner wall shows a strong development of ridges (fig. 2 *f*). The transition of the vasa deferentia into the testes is shown in fig. 2 *g*. The left testis does not become larger, whilst the right testis increases in size towards a more dorsal region (fig. 2 *h*).

The vasa deferentia of the specimen on *Metopograpsus quadridentatus* from Halmahera are wide, their inner wall has a few ridges only (fig. 2 *i*). The chitinous tube which forms the connexion of the left vas deferens with its testis is shown in fig. 2 *j*. In a more dorsal region the two testes become considerably larger, then they have approximately the same size (fig. 2 *k*).

In the specimen on *Metopograpsus quadridentatus* from Japan the vasa deferentia are wide, their wall has comparatively few ridges only. The transition of the left vas deferens in its testis is shown in fig. 2 *l*, that of the right vas deferens in its testis in fig. 2 *m*. In this specimen the right testis remains decidedly smaller than the left.

The individual variation in the structure of the male genital organs in these specimens is of the same order as that found in specimens on *Pachygrapsus crassipes* (cf. BOSCHMA, 1936, figs. 2, 3). Here too the vasa deferentia may have many or few internal ridges, and the two testes may be of equal size or decidedly unequal. In SHIINO's specimen of *S. yatsui* on *P. crassipes* also one of the testes is much larger than the other (cf. SHIINO, 1943, fig. 8 A).

In all the specimens of which sections were made the colleteric glands appear to have a flattened shape. Their canal system never is very strongly branched, the branches never are exactly arranged in one row parallel to the surface of the gland; as a rule there are two or more rows. The maximum number of canals in one longitudinal section of the colleteric glands is 26 in the specimen on *Metopograpsus oceanicus* (fig. 3 *a*), 19 in the specimen on *M. messor* from Kaledupa (fig. 3 *b*), 23 in the specimen on *M. quadridentatus* from Japan (fig. 3 *c*, *d*), and 31 in the specimen on *M. quadridentatus* from Halmahera (fig. 3 *e*, *f*). These numbers nicely correspond with those found in specimens on *Pachygrapsus crassipes* (26 and 28, cf. BOSCHMA, 1936).

Fig. 4 shows the excrescences of the external cuticle of the mantle of three of the specimens, viz., that on *Metopograpsus quadridentatus* from Halmahera (fig. 4 *a*), that on *M. quadridentatus* from Japan (fig. 4 *b*), and that on *M. oceanicus* (fig. 4 *c*). The specimens on *M. messor* possess excrescences of the same kind, showing a similar degree of individual variation. In each of the specimens in some parts of the mantle there occur isolated spines or compounds consisting of two or three spines only. In other parts of the mantle the spines may form compounds of rather large groups. Moreover, there is a great deal of variation in the thickness of the spines and their manner of spreading. Also the height of the spines shows a considerable degree of variation. The excrescences as a whole vary in height from 30 to 65 μ.

In the specimens of *Sacculina yatsui* on *Pachygrapsus crassipes* the excrescences of the external cuticle also have a height of 30 to 65 μ (cf. BOSCHMA, 1936). In these specimens, however, there is a stronger tendency of remaining isolated than in the specimens on the various species of *Metopograpsus*. In the specimens from the East Indies the spines of the

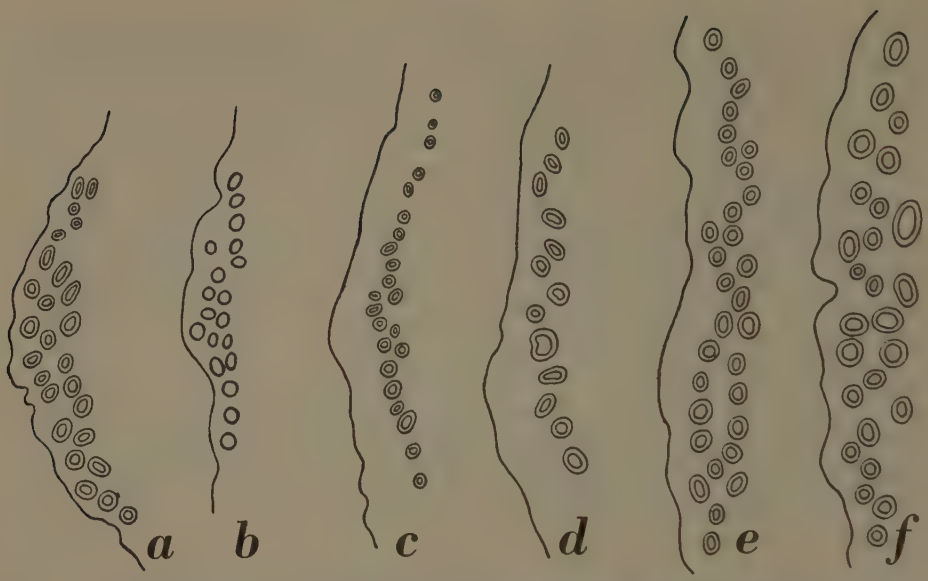

Fig. 3. *Sacculina yatsui,* longitudinal sections of the colleteric glands of four specimens. *a,* on *Metopograpsus oceanicus; b,* on *Metopograpsus messor* from Kaledupa; *c, d,* on *Metopograpsus quadridentatus* from Japan; *e, f,* on *Metopograpsus quadridentatus* from Dodinga. *a, b,* × 80; *c, d,* ×.45; *e, f,* × 90.

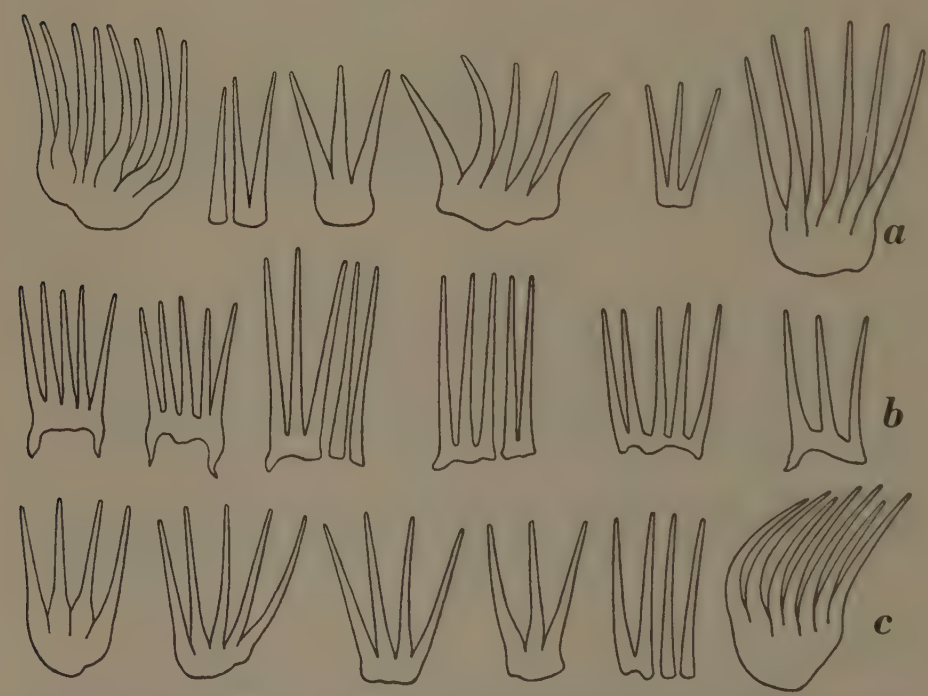

Fig. 4. *Sacculina yatsui,* excrescences of various parts of the external cuticle of three specimens. Upper row (*a*), on *Metopograpsus quadridentatus* from Dodinga; middle row (*b*), on *Metopograpsus quadridentatus* from Japan; lower row (*c*), on *Metopograpsus oceanicus.* × 530.

excrescences are more strongly united than in the Japanese specimen on *M. quadridentatus;* in the former often there is a well developed basal part whilst in the Japanese specimen the basal part never becomes very thick. In this respect the Japanese specimen on *M. quadridentatus* is extremely similar to the specimens on *Pachygrapsus crassipes,* until now known from the Japanese region only. SHIINO's figure (1943, fig. 8 B) of *Sacculina yatsui* on *Pachygrapsus crassipes* also shows spines which for the greater part are not united with other spines.

Sacculina yatsui is strikingly similar to *S. leptodiae* Guérin–Ganivet. It is, indeed, not easy to find characters by which the two species can be distinguished. In general the tendency for isolated spines is larger in *S. yatsui,* but some specimens of *S. leptodiae* also show this kind of excrescences (cf. BOSCHMA, 1948). The canal system of the colleteric glands is more strongly divided in *S. yatsui,* whilst the canals in the glands of *S. leptodiae* almost without any exception are distinctly arranged in one row. It is impossible to give distinct data to prove with certainty that the two species are different.

Sacculina plana Boschma

Takao, South Formosa, May 29, 1907, SCHAUINSLAND leg. (collection Zoological Museum Munich), 3 ex. on one specimen of *Metopograpsus messor* (Forsk.), dimensions of the parasite 8 × 5 × 2 mm.

The specimens are from the same locality as the type material (cf. BOSCHMA, 1933), which were parasites of the crab *Grapsus strigosus* (Herbst). The species seems to have a tendency for gregariousness, as among the type material there are four crabs with two parasites each, and the three specimens in the present material all occur on one crab. The specimens are more or less oval (fig. 5 *a*), not strikingly different from the type specimen (BOSCHMA, 1933, fig. 7 *i*). As in the latter the mantle opening slightly protrudes over its surroundings.

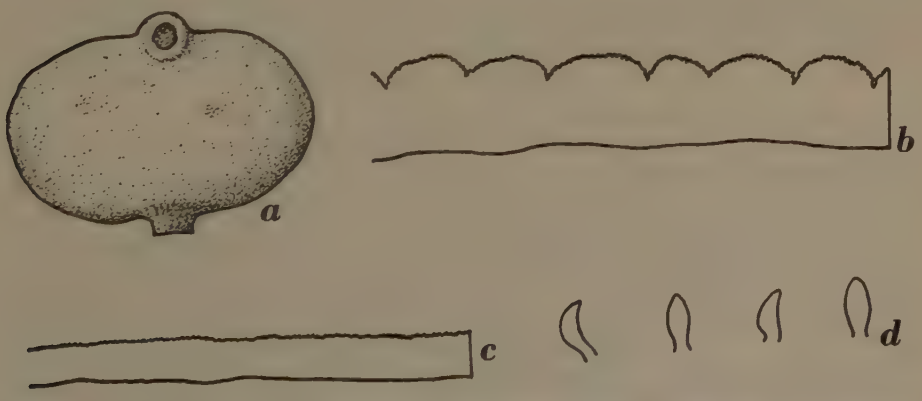

Fig. 5. *Sacculina plana,* specimen on *Metopograpsus messor.* *a,* left side; *b, c,* sections of the external cuticle; *d,* retinacula. *a,* × 5; *b—d,* × 530.

The male genital organs correspond with those of previously described specimens (cf. BOSCHMA, 1933, 1937). The vasa deferentia are narrow tubes (fig. 6 *a*), the testes have completely united to one large sac. The passing of the left vas deferens into this common testis is shown in fig. 6 *b*, that of the right in fig. 6 *c*. Towards the dorsal region the common testis still somewhat increases in size (fig. 6 *d*).

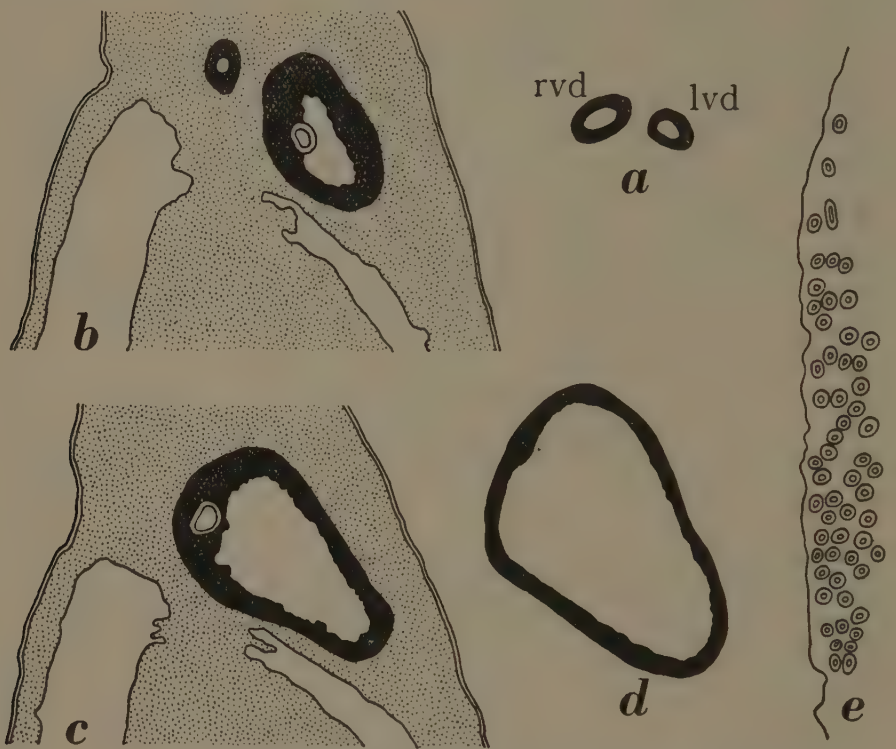

Fig. 6. *Sacculina plana*, specimen on *Metopograpsus messor*. *a*, transverse section of the ventral part of the vasa deferentia; *b*, posterior part of a longitudinal section showing the left vas deferens entering the testis; *c*, idem, showing the right vas deferens entering the testis; *d*, transverse section of the testis in its widest region; *e*, longitudinal section of one of the colleteric glands. *lvd*, left vas deferens; *rvd*, *right* vas deferens. *a*—*d*, × 45; *e*, × 80.

A longitudinal section of one of the colleteric glands in the region of its most strongly branched part shows 57 canals (fig. 6 *e*). In the type specimen (cf. BOSCHMA, 1937, fig. 70 *b*) there are 59 of these canals.

The external cuticle of the mantle does not possess excrescences. Its surface is not entirely smooth, but slightly rough (fig. 5 *b*, *c*). In various parts it is somewhat grooved so that a transverse section has a more or less undulating surface. The thickness of this cuticle varies from 8 to 30 μ.

The internal cuticle of the mantle bears retinacula which consist of a single spindle only. The length of these spindles is from 12 to 15 μ, barbs were not found. In the type specimen the spindles are slightly larger (18 μ).

Sacculina imberbis Shiino

Japan, HILGENDORF leg. (collection Zoological Museum Berlin), 1 ex. on *Metopograpsus quadridentatus* Stimps., dimensions of the parasite 9 × 4½ × 2 mm.

The type specimens were living on the crab *Pachygrapsus crassipes* (cf. SHIINO, 1943). The shape of the specimen on *Metopograpsus quadridentatus* (fig. 7 *a*) is very similar to that of the specimen figured by SHIINO (1943, fig. 1 I).

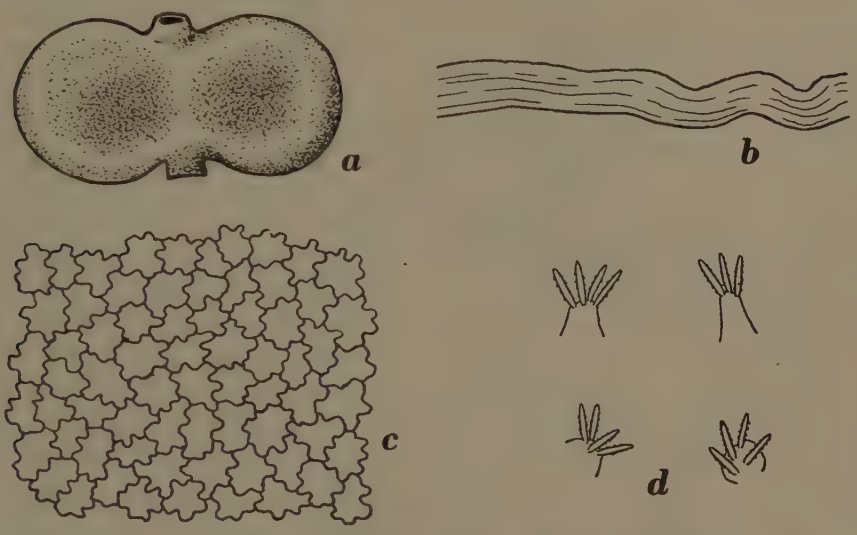

Fig. 7. *Sacculina imberbis*, specimen on *Metopograpsus quadridentatus*. *a*, left side; *b*, section of the external cuticle; *c*, surface of the external cuticle; *d*, retinacula. *a*, × 4.8; *b*—*d*, × 530.

The male genital organs correspond with those of the type specimen described by SHINO. They are found outside the visceral mass, in the region near the stalk (fig. 8 *e*). The vasa deferentia are narrow canals (fig. 8 *a*), their dorsal part has a distinct chitinous wall which penetrates into the ventral part of the testes (fig. 8 *b*). Towards the dorsal region the testes slightly increase in size (fig. 8 *c*, *e*) and gradually become narrower again (fig. 8 *d*).

The colleteric glands, which, as SHIINO remarks, are found near the anterior part of the visceral mass, are rather flat and possess a small number of canals only. In the present specimen this number is 14 in the most strongly branched region (fig. 8 *f*, *g*). Here the canals have a distinct chitinous wall, in SHIINO's specimen the chitin had not developed.

The external cuticle of the mantle (fig. 7 *b*, *c*) is rather thin (9—12 μ), it does not possess excrescences. Its surface shows small areas with an undulating contour (diameter of these areas 8 to 15 μ).

The internal cuticle of the mantle bears numerous retinacula which are more or less regularly distributed over its surface (fig. 7 *d*, here the retinacula are placed at a much shorter distance from each other than in

reality). Each retinaculum bears three or four slender barbed spines of a length of 9 to 12 μ.

Undoubtedly the present specimen belongs to *Sacculina imberbis* Shiino. In SHIINO's specimen the external cuticle did not show the areas which

Fig. 8. *Sacculina imberbis*, specimen on *Metopograpsus quadridentatus*. *a—d*, transverse sections of the male genital organs; *e*, posterior part of a longitudinal section showing the testes (approximately the same region as *c*); *f*, *g*, longitudinal sections of one of the colleteric glands. *a—e*, × 45; *f*, *g*, × 80.

occur in the present specimen. This difference is due to individual variation. Moreover, in many specimens of Sacculinidae which have these areas they are distinctly visible in certain parts of the mantle only. SHIINO did not find retinacula in his specimens. As they occur in the present specimen they undoubtedly also occur in specimens living on *Pachygrapsus crassipes*. But sometimes they are hard to find, especially when the internal cuticle is strongly adhering to the underlying tissues.

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Biology. — *Segmentatie.* By J. F. VAN BEMMELEN.

(Communicated at the meeting of September 24, 1949.)

In vele leer- en handboeken der Dierkunde, ook van den jongsten tijd, wordt van de Mollusken gezegd, dat zij ongelede dieren zijn. Zelfs wordt deze bewering veelal aan den aanhef der korte omschrijving van hun bouwtype geplaatst, wat meebrengt dat zij niet nader wordt toegelicht en dus als een vaststaand feit, dat geen verdere bewijsvoering nodig heeft, wordt voorgesteld. Vraagt men zich echter af, waarop deze overtuiging berust, dan rijst al spoedig het vermoeden, dat zij in de eerste plaats voortspruit uit het ontbreken van duidelijke uitwendige tekenen van geleding, terwijl toch de eenvoudige beschouwing der overgrote meerderheid van Gewervelde Dieren (dus van ongetwijfeld gelede dieren) ons leert, dat deze uiterlijke afwezigheid bij hen evengoed voorkomt. Daarbij komt, dat zodra men dieper op de zaak ingaat, men tot het inzicht geraakt, dat die afwezigheid slechts schijnbaar is. Niet alleen, dat een vermageringskuur de segmentale rangschikking der ribben en wervels door de huid heen zichtbaar maakt, maar ook bij normalen voedingstoestand vertonen tal van Gewervelden uit alle afdelingen (b.v. damherten, appelschimmels, gevlekte katten en panters, gestreepte tijgers en tal van Reptielen en Vissen) een regelmatige segmentale indeling der kleurentekening, overeenstemmend met die van het geraamte. Vooral blijkt de betekenis dezer waarneming, wanneer men door nauwkeuriger onderzoek tot het besef komt, dat alle niet-segmentale patronen der kleurentekening wijzigingen zijn van de segmentale, en dus uit deze ontstaan zijn. In 't bijzonder geldt dit voor de eenkleurigheid, zoals dit zo overtuigend te zien is bij geheel zwarte of bruine Zoogdieren (b.v. zwarte katten of paarden) bij welke vooral in de jeugd, de regelmatige gevlektheid als „kleur-in-kleur" doorschemert, en evenzeer bij de appelschimmels.

Voortgaande in dezen gedachtengang komt men vanzelf tot de voorstelling, dat ook de rangschikking der toegangsopeningen tot de zintuigen, de spijsverterings- en ademhalingsorganen aan kop en hals (dus de neusgangen, de oogkassen, de mondopening, de gehoorgangen en de overige kieuwspleten) op segmentatie berust. Hetzelfde geldt voor de ontwikkeling van het haarkleed der Zoogdieren, het veerkleed der Vogels, het schubbenkleed der Reptielen en Vissen, zoals dat o.a. zo duidelijk te zien is aan den staart van de Hagedis-staartvogel Archaeopteryx, waar iedere staartwervel een paar stuurpennen draagt. Ook het menselijk hoofd levert van die metamere rangschikking der haarbekleding een duidelijk bewijs in de opvolging der haarzomen op bovenlip, boven- en onderooglid, wenkbrauwbogen, bakkebaarden, kinbaard en (in enkele gevallen) keelbaard.

Dat ook de aanwezigheid van twee paar ledematen aan segmentatie mag

toegeschreven worden, blijkt duidelijk uit hun ontwikkelingsgeschiedenis, waarbij aanvankelijk aan een reeks van metameren een ledemaatsknop ontspruit, van welke echter alleen in schouder- en heupstreek een beperkt aantal zich verder ontwikkelt, terwijl de overige weer verdwijnen. Zo kon ik aantonen, dat aan den aanleg der voorpoten van de Hagedissen oorspronkelijk zeven metameren deelnemen, van welke slechts vijf zich handhaven. Dit laatste aantal komt dus overeen met dat der vingers, hetwelk blijkbaar het grondtal bij alle Longademende Gewervelden is, en mogelijkerwijze ook aan de vinnen der Kieuwademende ten grondslag ligt.

De onderstelling ligt dus voor de hand, dat ook op andere plaatsen zich ledematen zouden kunnen ontwikkelen, en uit dat oogpunt gezien vraagt men zich af of niet de tong in de kopstreek en de penis in de heupstreek als met elkaar vergroeide ledemaatsstralen te beschouwen zijn. Voor de penis leveren de copulatie-organen der mannelijke haaien en roggen (de z.g. driestaarters) het afdoende bewijs, voor de tong is de gepaarde aanleg bij het embryo eveneens een duidelijke aanwijzing, evenals de gespletenheid van de slangentong.

Misschien mag ook de neus als een paar met elkaar vergroeide, holle ledematen (tentakels) opgevat worden: bij den olifant fungeert zij nog als zodanig, terwijl bij sommige vleermuizen de neusgaten op afzonderlijke kegelvormige uitsteeksels staan.

Eenzelfde verband bestaat tussen de gepaarde en de ongepaarde vinnen der Vissen: ook de laatste ontstaan uit de vergroeiïng van twee spierknoppen der wederzijdse myotomen in het mediane vlak.

Beziet men uit dit oogpunt de Mollusken, dan merkt men op, dat sommige Longslakken twee paar tentakels hebben, waarvan het tweede paar de ogen draagt. Maar veel sprekender symptomen van geleding vertonen de Keverslakken (Chitonen) wier rugvlakte een reeks van zeven schelpplaten draagt, terwijl in hun mantelgroeven een groter of kleiner aantal van kieuwboompjes op een rij gerangschikt staan. Daar nu de Chitonen om allerlei redenen (o.a. hun zuiver tweezijdige symmetrie) als de oorspronkelijkst gebouwde Weekdieren mogen opgevat worden, is er veel te zeggen voor de onderstelling dat alle Mollusken zich uit metameer-ingedeelde voorouders ontwikkeld hebben, maar van deze oorspronkelijke segmentatie slechts enkele sporen hebben overgehouden. Als zulke sporen mag men opvatten: het dubbele aantal kieuwen, nieren en hartevoorkamers bij Nautilus, de enig overgebleven vertegenwoordiger der eens zo talrijke Ammonieten en Nautiliden. Dat de overige Inktvissen maar één stel van deze organen hebben, berust dus m.i. op achteruitgang in aantal, en hetzelfde verschijnsel vertoont zich in een verder gevorderd stadium in de orde der Slakken, waar de meer oorspronkelijke onder de Zeeslakken. zoals de Oorslak (Haliotis) nog één paar kieuwen en nieren hebben, waarvan bij de verder gedifferentieerde één lid van elk stel is geoblitereerd, en ten slotte bij de Landlongslakken ook de tweede kieuw.

De eerste aanleiding tot deze ingrijpende wijziging in de organisatie mag

wellicht gezocht worden in de oorspronkelijke gewoonte, die nog bij de meeste slakken (Gastropoden) behouden is gebleven, om zich met de buikvlakte vast te zuigen aan den zeebodem, zodat zich aan die oppervlakte de z.g. voet ontwikkelde. Ook de Chitonen doen dit, en ook bij enige hunner heeft dat grote invloed op hun segmentale organisatie gehad, getuige Chitonellus en Cryptochiton, wier schelpen geheel of gedeeltelijk in den huidspierzak verdwenen zijn. Een zodanige fixatie heeft bij alle organismen, dierlijke zo goed als plantaardige, altijd een ingrijpenden invloed op de organisatie gehad, waarvan de gevolgen zich nog steeds doen gevoelen, ook nadat de vasthechting al lang weder opgeheven werd. Zo kan de organisatie der Cephalopoden alleen begrepen worden als men aanneemt dat zij ontstaan is uit een toestand waarbij de voet nog als zuigschijf werkte, doordien deze functie kwam te vervallen toen de dieren tot de vrijzwemmende levenswijze overgingen en hun voetranden naar elkander toekromden, zodat zij een uitlaattrechter gingen vormen voor het ademwater dat langs de kieuwen stroomde. Daar deze waterstroom van achteren naar voren was gericht, veroorzaakte hij een terugstoot die de dieren achteruit deed zwemmen, waarbij de ledematen aan hun kop (de vangarmen) zich aaneenlegden tot een bundel die een bedriegelijke gelijkenis met een vissenstaart vertoont. Vandaar dat een school kleine Inktvisjes in een aquarium, als men ze tot zijn verbazing achteruit ziet zwemmen, den indruk maakt van werkelijke visjes, die deze ongewone manier van beweging hebben aangenomen, terwijl bovendien hun (schijnbaar) kopeind geen duidelijke ogen vertoont, die integendeel op den staart schijnen te zitten. Zodra een der diertjes zijn tentakels uitspreidt, wordt het raadsel opgelost. *Koppotige* Inktvissen (Cephalopoden) hebben dus wel degelijk ledematen, en dezelfde qualificatie mag men toekennen aan den penis van vele Slakachtigen (Gastropoden).

De eigenaardige kamering van de schelp der Ammonieten en Nautiliden mag m.i. eveneens als een segmentatie worden opgevat, die dan echter niet tot den eersten aanleg van het dier bepaald blijft, maar gedurende het gehele leven periodiek plaats vindt.

Ook het zenuwstelsel van sommige Mollusken vertoont duidelijke sporen van segmentalen bouw, in de overlangse buikstrengen, die door een reeks dwarscommissuren met elkaar verbonden zijn. En het zijn juist weer de oorspronkelijkste Gastropoden bij welke deze longitudinale buikstrengen worden aangetroffen, zodat men recht heeft te onderstellen dat zij bij de meer gedifferentieerde verwanten door achteruitgang zijn verdwenen.

De Mollusken zijn niet de enige dierklasse, waarvan zelfs in recente handboeken wordt beweerd, dat zij ongeleed zijn; hetzelfde wordt b.v. gezegd van de Brachiopoden, ofschoon toch naast de meerderheid der soorten, die één paar niertrechters bezitten, er een enkele voorkomt: (Rhynchonella) die twee paar zulke trechters bezit.

Het is wel opmerkelijk, dat deze organen bij andere duidelijk gelede dieren den naam segmentaalorganen hebben gekregen, daar zij bij hen in groter of kleiner aantal segmentsgewijs gerangschikt voorkomen. In aanleg

zijn zij excretie-organen, die met een trilhaartrechter in de lichaamsholte beginnen en daaruit met de afvalproducten der stofwisseling tevens de geslachtscellen naar buiten voeren. De lichaamsholte (coeloom) is aanvankelijk eveneens in afdelingen geplitst, maar kan ten slotte door het wegvallen der schotten tot een enkele ruimte worden herleid, waarin het darmkanaal met zijn aanhangselen aan vliezige scheilen (mesenteriën) is opgehangen. Secundair kan die ruimte bij de Zoogdieren weer in borst- en buikholte worden verdeeld door een spierschot (het middenrif), terwijl primair een van de oorspronkelijke schotten blijft bestaan als het hartezakje (pericardium). Bij het Lancetvisje (Amphioxus) blijft nog een reeks schotten behouden.

Dat ook het darmkanaal der Gewervelde dieren den invloed der segmentatie heeft ondergaan, blijkt behalve uit de reeks der kieuwzakken in de halsstreek uit de drie paar speekselklieren in de mondstreek, het paar longen in de borststreek, de oorspronkelijk gepaarde leveruitstulpingen in de buikstreek. Ook de alvleesklier mag als een oorspronkelijk gepaarde uitstulping beschouwd worden, en eveneens de blinde darm, die bij de Vogels nog dubbel is.

Het ongelede karakter van het eigenlijke spijsverteringskanaal is derhalve een gevolg van het ineenvloeien der compartimenten van de lichaamsholte door het wegvallen der dwarsschotten. Daardoor wordt het darmkanaal in staat gesteld om zich binnen die holte in kronkels te gaan leggen, waardoor de weg waarop de spijs met het secernerende en resorberende oppervlak van de darmwand in aanraking is, meer of minder wordt verlengd. Dat het darmkanaal oorspronkelijk recht toe recht aan van mond tot anus verliep, wordt ons nog door primitieve vissen aangetoond.

Van den aanvang af moet dit kanaal een doorlopende verbinding gevormd hebben tussen de achtereenvolgende segmenten, zij het ook, dat het binnen ieder metameer een paar zijdelingse uitstulpingen vormde, die ten minste in het voorste gedeelte door poriën of spleten naar buiten mondden (de kieuwspleten).

Dit doorlopende karakter deelt het darmkanaal met het centraal zenuwstelsel en de ruggestreng (chorda dorsalis), tot zekere hoogte ook met het vaatstelsel. Terwijl echter voor zenuw- en vaatstelsel de bewijzen van hun oorspronkelijke segmentale organisatie voor de hand liggen (men denke b.v. aan de ruggemerg- en hersenzenuwen, de sympathicus en de aortabogen) is dit voor de chorda niet zo vanzelfsprekend: de ontwikkeling van de wervelkolom maakt den indruk alsof de oorspronkelijk onverdeelde ruggestreng door de segmentsgewijs ontstaande wervellichamen en bogen plaatselijk wordt ingesnoerd en daardoor secundair wordt gesegmenteerd.

Maar als men de eerste ontwikkelingsstadiën van zenuw- en vaatstelsel nagaat, zijn ook deze aanvankelijk niet metameer verdeeld, maar overlangs doorlopend van kop naar staart: het centraal zenuwstelsel in den vorm van het medullairkanaal, het vaatstelsel als de primaire zijdelingse arteriestammen, die in de toekomstige halsstreek aan de buikzijde met elkaar ver-

smelten tot de hartbuis. Deze enkelvoudige aanleg wijst op het verband tussen één- en meercellige wezens: ook de Protozoën en Protophyten bezitten een kop- en een staartpool, vele hunner, zoals de Infusoriën, een mond en een anus, terwijl in het tussenliggende protoplasma een vaste baan zich aftekent waarlangs de spijs-vacuolen van ingestie- naar egestieporie worden gestuwd. Zulke eencelligen hebben dus een virtueel darmkanaal zonder permanent lumen, en deze vaste baan der spijsvacuolen kan zelfs een kronkelend verloop bezitten.

Den overgang van het één- tot het meercellige stadium kan men zich aldus voorstellen, dat bij de voortplantingsdeling van het eencellig organisme de deelstukken met elkaar in samenhang gebleven zijn en zich als één wezen zijn gaan gedragen, wat noodzakelijkerwijze tot functieverdeling moest leiden. De verdeling van de bevruchte kiem der meercellige organismen in deelstukken die met elkaar gaan samenwerken is dus een voortplantingsverschijnsel, wat ten overvloede bewezen wordt door de gevallen, waarin deze deelstukken weer van elkaar los laten en elk voor zich tot een volledig organisme uitgroeien.

In dezen gedachtengang voortgaande, komt men tot de voorstelling, dat ook de segmentatie een voortplantingsverschijnsel is. Het is toch duidelijk, dat de onderlinge ongelijkheid der metameren van een organisme moet ontstaan zijn uit onderlinge overeenkomst. Vergelijkt men b.v. de Insekten of de Kreeftachtige Dieren met de Gelede Wormen, dan kan men zich de eerstgenoemde wel uit de laatste ontstaan denken maar niet omgekeerd. Deze onderlinge gelijkheid der segmenten kan echter nooit volledig geweest zijn: het voorste en het achterste segment moeten van den aanvang af verschild hebben, zowel van elkaar als van de tussenliggende. Maar alle segmenten moeten oorspronkelijk volledig georganiseerd zijn geweest; zij moeten alle deel hebben genomen aan den opbouw van al de orgaanstelsels. Dit betekent, dat zij in aanleg compleet gebouwde organismen waren, die hunne zelfstandigheid konden hernemen door van elkander los te laten. Dit verschijnsel zien wij dan ook in allerlei afdelingen van het Dierenrijk, b.v. bij de Strobilatie der Kwallen, de Proglottiden der Lintwormen, de verdeling van vele Gelede Wormen door kopvorming in het midden der keten van hun segmenten, terwijl ook de vorming ener reeks van nieuwe organismen in den kiemstok der Salpen als een vorm van segmentatie kan opgevat worden.

Tegenover dit streven der metameren naar volledigheid en zelfstandigheid staat een even algemene drang naar verhoogde samenwerking onder toenemende mate van onderling verschil en verlies van volledigheid. Deze drang voert tot vergroeiïng van metameren, wat gepaard gaat met grotere of geringere vervaging van het gesegmenteerde karakter. Duidelijke voorbeelden daarvan levert de wervelkolom der Zoogdieren en Vogels, in de onderlinge vergroeiïng der heiligbeenwervels, de inkrimping der halswervels van de Walvissen onder verlies van hun wederzijdse bewegelijkheid, het staartbeen van den Mens en de mensvormige Apen, evenals dat

van de recente Vogels, in tegenstelling met den uit 24 vrije wervels opgebouwden staart van Archaeopteryx. Ook hierbij ontmoeten wij weer voorbeelden van terugkeer tot den vrijen toestand, in de afwezigheid van een gedifferentieerd heiligbeen bij de Walvissen, den minder geconcentreerden en gereduceerden toestand der staartwervels bij de Struisen en andere Loopvogels, waardoor een schijnbaar primitieve toestand ontstaat.

Vergelijken wij de wervelkolom der Slangen met die der Hagedissen, dan treft ons niet slechts de afwezigheid van een heiligbeen, maar ook de grotere overeenkomst der (met bewegelijke ribben uitgeruste) halswervels met de ruggewervels, die aan een meer oorspronkelijken toestand der eerstgenoemde doet denken, wat opnieuw een voorbeeld geeft van terugkeer tot een vroegeren toestand (Atavisme). Bij de Slangen gaat deze uniformiteit der hals-, rugge-, lenden- en heiligbeenwervels gepaard met een aanzienlijke vermeerdering in aantal, wat wijst op het vermogen der metameren om zich in onderdelen te splitsen, dus op een herhaling van het vermenigvuldigingsverschijnsel. Iets dergelijks moet ook ten grondslag liggen aan de onderverdeling van elk der metameren van het achterlijf der rupsen in acht ringen, die ook in hun beharing en vlekkentekening tot uiting komt.

Tot deze zelfde opmerkingen geeft de kleurentekening der achterlijfssegmenten van velerlei Insekten aanleiding, b.v. naast de rupsen en vlinders ook de wespen, vliegen, wantsen, kevers en glazemakers. Vooral de rugzijde der abdominaalsegmenten draagt een meer of minder gedifferentieerd patroon, en dit vertoont een duidelijk bipolair karakter, in zoverre als de schakering der kleuren langs den kopwaartsen rand verschilt van die langs den staartwaartsen. Aan de buikzijde zijn het de stippenparen op de pootloze segmenten, die op dezelfde plaatsen staan als de buikpoten op de wel van ledematen voorziene, en dus het bewijs leveren dat alle segmenten oorspronkelijk van een ledematenpaar voorzien waren, hetgeen door de ontwikkelingsgeschiedenis bevestigd wordt, aangezien bij tal van Insekten de embryonen op ieder metameer een potenpaar verkrijgen.

Bij Insekten treft ons nog een andere indeling van het lichaam, n.l. in kop, borststuk en achterlijf, en hetzelfde is het geval bij Spinnen (vooral Schorpioenen), Duizendpoten en Crustaceeën.

Op de tegenstelling tussen deze twee vormen van geleding heeft SIMROTH gewezen in zijn merkwaardig boek: Die Entstehung der Landthiere, en daarbij voorgeslagen om bij deze verdeling in drieën te spreken van macrosomieten, bij die in lichaamsringen van microsomieten. RAY LANKESTER, in zijn verhandeling Limulus an Arachnid heeft de termen Pro-, Meso- en Metasoma ingevoerd, omdat de benamingen Kop, Borststuk en Achterlijf bij toepassing op de Spinachtige Dieren tot misverstand aanleiding kunnen geven.

Maar waarop tot nu toe geen acht is geslagen, dat is het principiële onderscheid tussen beide vormen van indeling, naar mijn opvatting daarin bestaande, dat de mikrosegmentatie een voortplantingsverschijnsel is, terwijl de makrosegmentatie niet met vermenigvuldiging in verband staat. Beide

vormen van differentiatie zijn daarbij niet tot de klasse der Geleedpotige Dieren beperkt, maar komen door het gehele Dierenrijk voor, en kunnen ook in het Plantenrijk teruggevonden worden. Uitgaande van de onderstelling dat alle levende wezens oorspronkelijk bipolair en zygomorph georganiseerd zijn, dus een voor- en een achtereind, een rug- en een buikzijde, een rechts en een links bezitten, ziet men in dat de verdeling in pro-, meso- en metasoma niet anders is dan een verhoogde functionele differentiatie dezer drie lichaamsstreken, die van den aanvang af aan verschillende levensvoorwaarden moesten voldoen. Dat bij de naakte Amoeben de polariteit en de symmetrie schijnt te ontbreken, mag aan een terugwijken dezer eigenschappen worden toegeschreven; dat zij niet werkelijk afwezig zijn, blijkt zodra de naakte protoplasten zich met een pantser omgeven of zich gaan delen.

Deling is vermenigvuldiging: uit een enkelvoudig organisme ontstaan er twee of meer. Elke deling wordt ingeleid door een insnoering, die bij de dwarsdeling loodrecht op de lengte-as verloopt. Ter verklaring van andere deelrichtingen, zoals de overlangse, mag men aannemen, dat de polen van de kern zich verplaatsen kunnen. Maar de tegenstelling tussen kop- en staartpool moet zich altijd handhaven: isodiametrische rangschikking der bestanddelen, zoals bij de kristallen van het regulaire systeem, is bij het protoplasma niet mogelijk.

Niettegenstaande haar oorspronkelijkheid kan toch de tegenstelling tussen beide lichaamspolen zo gering zijn, dat er uiterlijk geen onderscheid te zien is. Dat is waarschijnlijk de reden, dat bij beschrijving en afbeelding van celketens zoals Spirogyra, van afgeronde organismen zoals vele stuifmeelkorrels, of van Flagellatenkolonies gelijk Volvox van de mogelijke aanwezigheid van polairen bouw geen gewag wordt gemaakt. Toch ben ik van mening dat deze bestaat, en door diepgaand onderzoek aangetoond zou kunnen worden. Bij Volvox is dit zelfs zonder meer duidelijk.

Dat ieder lid van een Spirogyradraad als een zelfstandig organisme mag beschouwd worden blijkt bij de bevruchting van elk hunner door een lid van een anderen draad. Dit leidt tot de vraag in hoeverre bij gesegmenteerde organismen van hogere differentiatie, de onderscheidene metameren zich nog als zelfstandige wezens kunnen doen gelden, zonder zich uit het complex los te maken. Bij de Lintwormen en de kiemstok der Salpen, waar deze loslating wel plaats vindt, baart de oorspronkelijke samenhang geen bezwaar, maar is deze ook veel minder innig dan bij die vormen van segmentatie, waarbij de aaneengeschakelde metameren een aanzienlijk deel hunner volledigheid en ingevolge daarvan hun zelfstandigheid, verloren hebben. Toch kan ook bij enkele dezer meer geconsolideerde organismen een willekeurig lid der segmentenketen zijn zelfstandigheid hernemen en aan zijn voorrand een kop gaan vormen, wat op den achterrand van het voorgaande segment in zoverre van invloed is, dat deze zich tot een staarteind afsluit.

Deze kopvorming in de reeks der metameren kan zich zelfs herhalen,

zodat een keten van onderling samenhangende individuen, elk bestaande uit een zeker aantal metameren, ontstaat, die dan ten slotte van elkaar loslaten en dus zelfstandige vrijzwemmende wormen worden (Paloloworm). Dit vermogen om zich in stukken te verdelen, die een zelfstandig bestaan gaan voeren, is zelfs niet alleen aan gesegmenteerde organismen gebonden, maar komt eveneens bij ongelede wormen voor en kan daarbij kunstmatig door verwonding worden opgewekt (Planaria).

In een werkje van VITUS GRABER over Insecten, waarmede ik in 1875 kennis maakte, betoogt de schrijver dat de segmentatie dezer dieren een gevolg is van de ontwikkeling van het chitine-pantser, dat noodzakelijkerwijze in ringen moest verdeeld worden, die door dunne en plooibare huidstukken bleven samenhangen, om bewegelijkheid van het lichaam mogelijk te maken. In een recent handboek der Dierkunde (STEMPELL, Zoologie, 1935) wordt diezelfde verklaring van het ontstaan der metamerie gegeven. Maar als men zich aan de hand daarvan den gang van zaken tracht voor te stellen, is men gedwongen aan te nemen, dat de verstijving der chitine-bekleding reeds in segmentsgewijs-verdeelde opeenvolging plaats moest grijpen; met andere woorden dat het lichaam reeds in segmenten verdeeld was, voordat zich om ieder segment een verdikte ring vormde, wat betekent, dat de mechanische verklaring berust op verwarring van oorzaak met gevolg.

In tegenspraak met haar is ook het feit, dat verharding en verdikking van het chitine pantser meestal gepaard gaat met vermindering der bewegelijkheid van de segmenten ten opzichte van elkaar, zoals wij dit zien bij de vergroeiïng van de drie delen van het borststuk der Insekten. Ook gaat die verklaring in 't geheel niet op bij de vraag hoe het komt dat de inwendige delen hun oorspronkelijken gesegmenteerden aanleg weer verliezen, zodat b.v. de nieren der Zoogdieren daarvan zo goed als niets meer vertonen. Als men zegt, dat zodanige vervormingen teweeggebracht worden enerzijds door obliteratie van een deel der segmenten, anderdeels door concentratie van de rest, wordt daarmede alleen de loop van het verschijnsel beschreven, zonder dat de oorzaak ervan wordt verklaard. Evenmin als voor andere levensverschijnselen geeft voor de segmentatie de aanduiding van een zuiver mechanische invloed een afdoende verklaring.

Leiden, Augustus 1949.

Summary.

In modern zoological textbooks several animal types, e.g. Molluscs, Brachiopods and even Tunicates are circumscribed as unsegmented, on account of the absence of definite external features of metameric division, though the investigation of their internal organisation procures proofs of the contrary.

Two kinds of segments may be distinguished, called macro- and microsomites, according to the nomenclature of SIMROTH. The former stand in

connection to the composition of the body in three regions, which are best designated after RAY LANKESTER's proposition as pro-, meso- and metasoma. The latter on the contrary occur in the most varying number and in every degree of differentiation, from completely organised individuals to mere vestiges. The division into microsomites (metameres) may be considered as a form of propagation (multiplication) which in several falls is proved by their getting free from each other and behaving as separate individuals (Proglottids of Bandworms, Ascidiozoids of Pyrosoma, zoöids of the stolon of Salpa, Ephyres of the Strobila of Jelly-fiches-fission of Chaetopods).

In ordinary segmentation which does not lead to isolation of the separate metameres, their independent character still shows itself in the Zygomorphic arrangement of their skinpattern.

Résumé.

En maintes traités de Zoologie les Mollusques et plusieurs autres types d'animaux, comme les Brachiopodes et même les Tuniciers, sont décrites comme non-articulé, à cause de l'absence de traces externes de division en segments.

L'étude de leur organisation interne et surtout de leur embryologie fait découvrir des restes indubitables d'une division en métamères. Deux formes de division se laissent distinguer, nommées macro- et microsomites d'après la proposition de SIMROTH. Les premiers ne sont autre que des parties differentiées des trois régions du corps de tout être vivant, des plantes aussi bien que des animaux. Ce sont les Pro-, Meso- et Metasoma selon la nomenclature de RAY LANKESTER. Les derniers au contraire, quoique aussi bien propres à tous les organismes, se rencontrent en nombre excessivement variable. En outre il peut exister une très-grande différence entre les segments d'un même organisme, surtout quand ils s'unissent par concrescence et forment des complexes, en perdant leur caractère d'individus indépendants. Le grade de leur développement peut monter jusqu'à la condition d'un organisme complêt, ou bien se baisser jusqu'à devenir presque méconnaissable. Selon mon opinion cette division en microsomites (appelés aussi segments, métamères, annaux) doit être considérée comme une forme de propagation, c'est à dire comme une conséquence de la tendance de chaque organisme vivant à se multiplier. Exceptionellement cette tendance est assez forte pour contraindre les membres de la chaine de métamères à se détacher et commencer une vie indépendante, dans le plupart des cas ils restent en connection plus ou moins intime. Comme exemples d'isolation finale peuvent être citées les Proglottides des Cestodes, les Zooïdes du Stolon des Salpes les Ascidiozoïdes de Pyrosoma les Ephyres du Strobila des Méduses la reproduction asexuelle de différentes Vers annelides (Palolo). Le caractère polair et bilatéral de chaque métamère se manifeste souvent dans le patron des couleurs du peau.

Zusammenfassung.

In vielen zoologischen Handbüchern werden Tiertypen wie Mollusken, Brachiopoden und selbst Tunicaten als ungegliedert gekennzeichnet, weil äussere Merkmale der Metamerie zu fehlen scheinen. Bei Untersuchung des inneren Baues und der Entwicklungsgeschichte lassen sich jedoch ihre Spuren nachweisen. Es giebt zwei Arten der Segmentation: die in Macro- und die in Microsomiten (nach SIMROTH's Nomenclatur). Die Macrogliederung beruht auf der Verteilung des Körpers in Vorder-, Mittel- und Hinterleib (Pro-, Meso- und Metasoma nach RAY LANKESTER). Diese war meiner Ansicht nach allen lebenden Geschöpfen vom Anfang an eigen. Die Micro-Einteilung dagegen, die eigentliche metamerische Verteilung in Segmente, kommt in den verschiedensten Zahlen vor, und daneben auf allen Stufen der Ausbildung der einzelnen Metamere, von derjenigen vollständiger Organismen bis zu fast unkenntlichen Spuren segmentaler Einteilung. Deshalb betrachte ich die Metamerie als eine Form der Fortpflanzung, wobei die endgültige Loslösung der einzelnen Abschnitte sich nur in einigen wenigen Fällen behauptet hat (Proglottiden der Bandwürmer, Ephyren der Quallen-Strobila, Zoöiden der Stolonbildenden Salpen, Actiniozoöiden der Pyrosomen, die Abschnitte des Palolowurmes und mehreren anderen Borstel-Würmer. Die zygomorphe Organisation der einzelnen Metamere verrät sich noch sehr oft in ihrer Farbenausstattung, die oft einen deutlichen bipolaren Character trägt.

Chemistry. — *The Anterior Pituitary Lobe and the Amino-Acid content of the Blood III.* By G. VAN WIERINGEN and S. E. DE JONGH. (Pharmacological laboratory, University Leyden.) (Communicated by Prof. P. J. GAILLARD.)

(Communicated at the meeting of September 24, 1949.)

In a previous paper we [1]) stated that it was observed (in experiments on adult rats of which the kidneys and adrenals were tied or removed) that pituitary extracts rich in growth hormone caused an increase after two hours in the percentage of amino-acids in the blood. We considered a primary slowing down of deamination in the liver as a probable interpretation of the foregoing. Other authors (N. K. SCHAFFER and M. LEE [2]); M. REISS, c.s. [3]); K. E. PASCHKIS [4]); J. FRAENKEL–CONRAT c.s. [5]) described in normal rats a fall similar to that which we [6]) found in eviscerated animals, and also in non-eviscerated, hypophysectomized rats:

In hypophysectomized rats either 50 tail growth units of growth hormone (1U = 350 γ) were injected intravenously, or the appertaining solvent (0.5 ml.).

At the commencement of the experiment and after two hours, blood was taken from the tail to determine the percentage of amino-acids. In control animals the percentage decreased slightly or not at all, but in growth hormone-animals there was a decided fall. (Table I).

TABLE I.

Hypophysectomized rats. Amino-acid content of the blood in mg % N.

Pituitary extract			Control solution		
Initial value	Final value	Difference	Initial value	Final value	Difference
4.42	3.85	0.58	4.66	4.74	–0.08
4.66	3.86	0.80	5.10	4.70	0.40
5.36	4.00	1.36	6.30	5.90	0.40
5.92	4.50	1.42	6.50	6.14	0.36
5.84	4.50	1.34	6.24	5.70	0.54
5.24	4.14	1.10 ± 0.17	5.76	5.44	0.32 ± 0.11

$$\frac{\text{Difference}}{\text{Stand. dev.}} = \frac{0.78}{0.20} = 3.9$$

If our former findings, like those of the above-mentioned authors, have any significance the kidneys or suprarenals must nullify the results of the lowered deamination in the liver.

The adrenal cortex promotes deamination in the liver. The kidneys can themselves effect deamination, though perhaps through another mechanism than the liver.

In order to choose between these two possibilities, we continued our

experiments with (*A*) adrenalectomized animals; (*B*) animals with tied kidneys.

In series (*A*) the left adrenal was removed and the right one tied prior to the experiment. In series (*B*) the kidneys were previously tied. The further procedure was similar to that in the foregoing experiment.

The result are shown in tables II and III.

TABLE II.

Adrenalectomized rats. Amino-acid content of the blood in mg % N.

Pituitary extract			Control solution		
Initial value	Final value	Difference	Initial value	Final value	Difference
9.0	7.3	1.7	6.4	5.9	0.5
6.4	5.3	1.1	5.9	5.5	0.4
5.8	5.6	0.2	6.5	4.9	1.6
6.3	5.6	0.7	7.0	5.5	1.5
6.2	4.9	1.3	8.1	6.1	2.0
6.74	5.74	1.00 ± 0.26	6.78	5.58	1.20 ± 0.32

$$\frac{\text{Difference}}{\text{Stand. dev.}} = \frac{0.2}{0.41} = 0.49$$

TABLE III.

Rats with kidneys ligated. Amino-acid of the blood in mg % N.

Pituitary extract			Control solution		
Initial value	Final value	Difference	Initial value	Final value	Difference
3.6	4.5	–0.9	6.1	3.9	2.2
4.4	3.3	1.1	6.1	3.6	2.5
5.4	4.8	0.6	5.9	2.9	3.0
6.9	6.0	0.9	6.6	5.9	0.7
5.9	5.0	0.9	5.9	4.9	1.0
5.2	4.5	0.7	6.1	4.5	1.6
			6.9	4.5	2.4
5.2	4.7	0.5 ± 0.30	6.1	4.3	1.8 ± 0.32

$$\frac{\text{Difference}}{\text{Stand. dev.}} = \frac{1.4}{0.437} = 3.2$$

It follows, therefrom, that neither the previously mentioned (relative) increase of the percentage of amino-acid nor a decrease is produced by growth hormone in adrenalectomized animals, but the rise takes place in nephrectomized animals. The kidney prevents the rise. It is presumed that (in the short time of our experiments) the kidney acts in a compensatory way if deamination in the liver is retarded. Since a fall in the amino acid content of the blood was observed not only in normal but also in hypophysectomized animals, whose adrenals certainly did not function at a high rate, it is difficult to assume an important influence of the adrenals in this respect.

From these experiments no certainty is derived for the hypothesis that the growth hormone retards the deamination in the liver. Unpublished observations by our co-worker, MAASSEN, on liver tissue in vitro have, however, confirmed this hypothesis.

It should be pointed out at this stage that it may be supposed, on the basis of the above findings, that the growth hormone causes (at the very least) no retardation of deamination in the kidney. The investigation is being continued along these lines.

Summary.

In rats without adrenals and kidneys growthhormone causes a rise in the amino acid content of the blood whereas in intact animals a decrease occurs.

The same rise was seen in rats with tied kidneys. Extirpation of the adrenals merely prevented the fall.

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Chemistry. — *The Sugar Excretion of Rats with Severe Diabetes, induced by Alloxan under the Combined Influence of Insulin and Growth Hormone.* By G. VAN WIERINGEN and S. E. DE JONGH. (Pharmacological laboratory, University Leyden.) (Communicated by Prof. P. J. GAILLARD.)

(Communicated at the meeting of September 24, 1949.)

Introduction.

The growth hormone, administered in large quantities, is now generally considered to possess diabetogenic potency. This is also the case in animal species, e.g. the rat, with a pancreas capable of great resistance, as shown in previous work in this laboratory [1]). It was shown that sugar excretion of rats with slight alloxan diabetes increased when the growth hormone was administered. Rats which had been injected with alloxan but had not become diabetic, did not develop diabetes with growth hormone, probably because the pancreas was too resistant to *both* substances. Severe alloxan diabetes did not become worse through growth hormone. Was this because sugar excretion had reached its maximum? We shall try to answer this question.

Method.

Four adult rats which, after one alloxan injection of 75 mg/kg, excreted 3 or more g. glucose per day, were regularly observed during four days. In two the sequence was as follows: (*a*) control period, (*b*) insulin period, (*c*) control period, (*d*) growth hormone + insulin period, (*e*) control period. In two others the periods (*b*) and (*d*) were interchanged. 2×5 IU of insulin, 2×10 rat tail units of growth hormone, (1 U in 350 γ) were administered daily. It was ascertained whether sugar excretion was diminished more by insulin alone than by insulin plus a (diabetogenic) dose of growth hormone.

Results.

The results are given in tables which show that the growth hormone did not have a diabetogenic action. The average fall caused by insulin is

$$\frac{(3.9 - 1.3) + (4.6 - 1.0) + (5.1 - 1.1) + (5.3 - 0.8)}{4} \text{ g.} = 3.7 \text{ g.}$$

with insulin + growth hormone the average fall was

$$\frac{(4.8 - 1.4) + (5.3 - 1.2) + (4.3 - 1.05) + (4.7 - 1.25)}{4} \text{ g.} = 3.5 \text{ g.}$$

DAILY SUGAREXCRETION IN GRAMS: ALLOXAN TREATED RATS.

Rat I		Rat II		Rat III		Rat IV	
3.8	Contr.	4.3	Contr.	4.5	Contr.	5.4	Contr.
4.4		5.4		5.2		5.9	
3.4		4.3		4.7		5.9	
3.5		4.4		4.3		4.7	
4.2		4.6		5.4		4.6	
Av.: 3.9		Av.: 4.6		Av.: 4.8		Av.: 5.3	
1.6	Insul.	2.1	Insul.	2.8	Insul. + Growthh.	2.0	Insul. + Growthh.
1.2		1.2		1.6		1.4	
2.0		0.5		0.9		1.0	
0.4		0.2		02		0.5	
Av.: 1.3		Av.: 1.0		Av.: 1.4		Av.: 1.2	
2.7	Contr.	3.5	Contr.	4.1	Contr.	3.9	Contr.
3.6		4.6		4.7		5.0	
5.2		5.2		5.7		6.5	
5.7		5.6		5.8		5.7	
Av.: 4.3		Av.: 4.7		Av.: 5.1		Av.: 5.3	
1.3	Insul. + Growthh.	2.5	Insul. + Growthh.	1.8	Insul.	1.4	Insul.
1.4		1.0		2.0		1.2	
0.6		0.4		0.5		0.3	
0.9		1.1		0.0		0.4	
Av.: 1.05		Av.: 1.25		Av.: 1.1		Av.: 0.8	
1.3	Contr.	3.8	Contr.	2.6	Contr.	4.2	Contr.
3.3		4.5		3.9		5.5	
4.1		5.7		6.0		6.7	
3.8		5.0		5.9		4.6	
Av.: 3.1		Av.: 4.75		Av.: 4.6		Av.: 5.25	

The increase on suspension of insulin was

$$\frac{(4.3-1.3)+(4.7-1.0)+(4.6-1.1)+(5.25-0.8)}{4}\ \text{g.} = 3.5\ \text{g.}$$

Suspension of insulin + growth hormone:

$$\frac{(5.1-1.4)+(5.3-1.2)+(3.1-1.05)+(4.75-1.25)}{4}\ \text{g.} = 3.3\ \text{g.}$$

The total average sugar excretion per day was:

for all normal periods: 4.6 g.
for all insulin periods: 1.05 g.
for all insulin + growth hormone periods: 1.2 g.

Growth hormone did have no more influence on animals with severe diabetes, whose sugar excretion was greatly suppressed by insulin administration, than on rats with severe alloxan diabetes not treated with insulin. The negative results previously obtained were not due solely to a possible maximum sugar excretion.

Discussion.

The simplest interpretation of our data would be, without doubt, a denial of the supposed diabetogenic effect of growth hormone in the rat. Our previous investigations do not favour this view, nor does the earlier work of MARX and others [2]), who worked on partially depancreatized rats.

Taking this into consideration we must assume that the growth hormone can do no more than give the results previously obtained with alloxan. On good grounds YOUNG [3]) presumed that the growth hormone, impedes carbohydrate utilization through a decrease of sensitivity to insulin. It follows that alloxan should also do this and to such a degree that there is nothing left for the growth hormone. Few objections can be raised against this assumption. It is generally known (it appears also from our data) that very large amounts of insulin are required to make an alloxan-diabetic animal sugar-free. An alloxan animal requires much more insulin than an animal without a pancreas [4]); this is ascribed to some factor in the pancreas. The idea that alloxan does more than merely restricting insulin production [5]) is gaining ground. We conclude that our negative results are due to a sugar utilization which has already been impeded to the highest degree by alloxan.

Summary.

Even when the sugar excretion in rats with severe alloxan diabetes is kept low by insulin, the growth hormone is not capable of increasing this (in contradistinction to the sugar excretion in rats with slight diabetes).

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Chemistry. — *The Relations between Growth and Insulin.* By G. VAN WIERINGEN and S. E. DE JONGH. (Pharmacological laboratory, Leyden.) (Communicated by Prof. P. J. GAILLARD.)

(Communicated at the meeting of September 24, 1949.)

Introduction.

The well-known investigations of YOUNG [1]) showed that diabetes (produced by the growth hormone) and body growth are mutually exclusive in the dog. Consequently insulin is considered indispensable for the action of the growth hormone. On the other hand, GAARENSTROOM and DE JONGH [2]) showed that growth hormone can still produce distinct growth in the hypophysectomized alloxan-diabetic rat; there are good grounds (VAN WIERINGEN [3])) for believing that such animals do not produce insulin. The aforementioned controversy might be ascribed to a difference between dog and rat, but it is also possible to argue as follows. YOUNG's dog had diabetes and lacked insulin. The rats of GAARENSTROOM and DE JONGH also lacked insulin but did not show any signs of diabetes, as the pituitary gland had been removed. The diabetic state would then be the decisive factor, not the lack of insulin.

It seemed to us of great interest to investigate whether a growth-hormonal influence on the protein metabolism could also be shown in rats with a pancreas damaged by alloxan. For that purpose we referred back to an investigation of GAARENSTROOM and KRET [4]) who showed in hypophysectomized rats on a protein-free diet that N-retention was caused by administration of growth hormone. This experiment was repeated on animals which previously had been treated with alloxan and had reacted to this with a severe diabetes, the signs of which (as expected) disappeared after hypophysectomy.

Method.

Young rats, weighing 120—150 g. were injected with 75 mg/kg. alloxan. From these animals were selected those with a sugar secretion of > 2.4 g. per day. These were hypophysectomized and (after a day on ordinary food) put on a protein-free diet (64 gr. rice starch, 6 g. olive oil, 4 g. MCCOLLUM-salt mixture and 45 g. water) which they ate and libitum (on an average nearly 10 g. corresponding to about 5 g. dry substances). The urine of the first day of experiment was rejected. After that (1st period) during three days urine was collected for N-determination. Subsequently growth hormone was injected (20 rat tail units per day, $1\ U = 350\ \gamma$) for four days into a certain number of these animals; the urine was collected for the last three days (2nd period). The groups of experimental and control animals were then exchanged (3rd period).

NITROGEN EXRETION OF ALLOXAN-TREATED HYPOPHYSECTOMIZED RATS.

Series	Bodyweight	1st period	2nd period		3rd period	
		mg N	mg N	%	mg N	%
A	132	198	97	44	65	33
	128	162	100	62	81	50
	132	190	115	60	85	45
	120	143	90	63	78	54
	138	185	124	67	131	71
	131	126	121	96	92	73
Av.:		167 contr.	108	65 contr.	89	54 contr.
B	148	177	88	50	110	62
	140	178	77	43	64	36
	140	194	80	41	66	34
	127	201	64	32	69	34
	132	152	47	31	92	60
	120	144	60	42	123	85
Av.:		174 contr.	69	40 growthhorm.	87	52 contr.
C	133	204	102	49	64	31
	160	186	118	63	63	34
	140	160	82	51	44	27
	157	195	123	63	43	22
	139	216	67	31	74	34
	135	222	155	70	71	32
Av.:		197 contr.	108	55 contr.	60	30 growthhorm.

Results.

The results are given in tables which show in all cases the excreted amount of N in urine in milligrams and in percentages of the values in the first period.

Group (*A*) remained untreated throughout and showed first a rapid and later a slow fall of the N- excretion; group (*B*), which received growth hormone in the second period, also reached its lowest average in N-excretion in this period. Group (*C*) which received growth hormone in the third period then gave the lowest mean value of all groups during all periods.

The average N- excretion in the second period of Group (*B*) (percentage column) differs significantly from that of the groups *A* and *C*; in the third period the N- excretion of Group *C* differs significantly from those of *A* and *B* ($P < 0.01$ in both cases).

Discussion.

With a protein-free diet (i.e., while growth is excluded) the pituitary extract used is capable of causing N- retention also in almost complete absence of insulin. It would be unreasonable to ascribe the effect to any other factor than the growth hormone. We believe that the growth of the normally fed alloxan rat can be explained in this way.

We [3]) already knew that the fall of the percentage of amino acid of the blood, which follows acutely on one single growth-hormone injection in the eviscerated animal, is definitely bound up with insulin, or at any rate fails to occur in alloxan animals. The interference of growth hormone in protein metabolism is dual! We [3]) had for that matter already found an indication in another way, and proposed as a preliminary interpretation (in addition to a displacement of the blood amino-acid body protein equilibrium) a primary fall of the deamination in the liver.

Summary.

In hypophysectomized rats on a protein-free diet a pituitary extract, rich in growth hormone, restricts the excretion of nitrogen with the urine. The same occurs when the animals have been previously brought into a severe diabetic condition by means of alloxan.

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Mathematics. — *A new characterisation of spheres of even dimension* [1]. By HSIEN-CHUNG WANG. (Communicated by Prof. L. E. J. BROUWER.)

(Communicated at the meeting of May 28, 1949.)

It is the aim of this paper to give a new characterisation of spheres of even dimension. We shall show that a simply-connected manifold is an even sphere if and only if its EULER characteristic is equal to two and it admits transitively a compact transformation group R. In the course of the proof, all the possible groups R are eventually determined. We find that R is either the orthogonal group or the CARTAN's exceptional group G_2, and that G_2 presents only when the manifold is six-dimensional. In an interesting paper [5], MONTGOMERY and SAMELSON have shown that the only compact transitive transformation group of a sphere S^{2n} of $2n$ dimension is the orthogonal group when $2n \geq 114$. By an entirely different method, we fill the gap they left. Furthermore, as an incident result of our discussions, we obtain the first four homotopy groups of the exceptional group G_2.

1. The chief weapon used in this paper is a finite group associated with a connected compact LIE group. This finite group has been fully discussed by various authors. In this section, we shall give a brief sketch of STIEFEL's results [7, §§ 2, 3] which will be used later.

Let R be a connected compact LIE group of dimension r and rank l. All the maximal toral subgroups of R have the same dimension l. Choose one of them, say T. Each normaliser a of T induces an automorphism φ_a: $t \to ata^{-1}$ $(t \in T)$ of T. All such automorphisms form a finite group which, up to an isomorphism, depends only on R and not on the particular choice of T. We shall denote it by $\Phi(R)$.

Let $U(e)$ be a small neighbourhood of the identity e of R such that it is covered by the canonical coordinates $\xi_1, \xi_2, \ldots, \xi_r$ of the first kind. These coordinates define an r-dimensional tangent euclidean space E^r of R. Each inner automorphism

$$x \to bxb^{-1} \qquad (x \in R)$$

of R indues a linear transformation S_b of the tangent space E^r which we call the adjoint linear transformation. Now let us consider the maximal toral subgroup T. Its tangent space E^l is a linear subspace of E^r and is l-dimensional. Evidently, the adjoint linear transformation S_a of each normaliser a of T leaves E^l invariant, and hence induces a linear trans-

[1] The present paper is the revised form of the second part of the author's Ph. D. thesis accepted by Manchester University, 1948.

formation φ'_a of E'. All these φ'_a form a finite linear group $\Phi'(R)$ isomorphic with $\Phi(R)$.

In the linear space E', there are m pencils of parallel hyperplanes, called *singular hyperplanes*, where

$$r = 2m + l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.1)$$

Through the origin there pass *exactly* m singular hyperplanes [3, p. 67]. Each transformation φ' of $\Phi'(R)$ carries singular hyperplanes to singular hyperplanes. Furthermore, we have [7, p. 363].

(1.2) *Let P be a point in E'. If P is left unaltered by a transformation $\varphi' \varepsilon \Phi'(R)$ other than the identity transformation, then there exists a singular hyperplane passing through both P and the origin.*

It is well-known that compact simple LIE groups fall into four main classes A_k, B_k, C_k, D_k $(k = 1, 2, \ldots)$ and five exceptional cases G_2, F_4, E_6, E_7, E_8 where the lower index denotes the rank. In fact, each of the above represents a class of locally isomorphic connected groups among which one is simply-connected and one without centre. In what follows, we shall occasionally use the term "an A_k $(B_k, C_k, \ldots, E_8)$" which simply means any group of the class A_k $(B_k, C_k, \ldots, E_8)$. The finite groups Φ associated with these simple groups have been completely determined.

For any connected compact LIE group R, let us denote by $o(R)$ the order of the finite group $\Phi(R)$. Then [9]

$$\left.\begin{aligned} &o(\text{toral group}) = 1,\ o(A_k) = (k+1)!,\ o(D_k) = k! \cdot 2^{k-1}, \\ &o(B_k) = o(C_k) = k! \cdot 2^k,\ o(G_2) = 12,\ o(F_4) = 3^2 \cdot 2^7, \\ &o(E_6) = 6! \cdot 3 \cdot 2^6,\ o(E_7) = 9! \cdot 2^3,\ o(E_8) = 10! \cdot 3 \cdot 2^6. \end{aligned}\right\} \quad (1.3)$$

2. A topological space W is called *homogeneous* if it admits transitively a topological group R of transformations. Let q be a point of W. All the transformations of R which leave q invariant form a closed subgroup L of R called the *isotropic subgroup*. The space W can be regarded as the space R/L of left cosets. R is said to be *effective* (*almost effective*) on W if only the identity (only a finite number of elements) of R preserves every point of W. Suppose that R is not effective on W. Then the elements of R which leave unaltered each point of W form an invariant subgroup I of R. The factor group R/I then acts effectively and transitively on W. Thus without loss of generality of the homogeneous space, we can assume that it admits an effective transformation group.

(2.1) *Let W be a homogeneous manifold of a connected compact group. If W has non-vanishing* EULER *characteristic, then it admits, transitively and almost effectively, a connected, simply-connected compact semi-simple* LIE *group R.*

Proof. By hypothesis, W admits, effectively and transitively, a connected compact group R'. Since W is locally euclidean and R' is

compact, it follows that R' must be a LIE group [10, § 7]. Moreover, the EULER characteristic of W does not vanish so that R' is simi-simple and has no centre [8, (2.1), (2.2)] [2]. Therefore the universal covering group R of R' is simply-connected, compact and semi-simple and has only a finite number of centres [6, p. 271]. This group R acts transitively on W in the natural manner. It is easy to see that an element b of R leaves unaltered every point of W if and only if b is a centre of R. Hence R acts almost effectively on W. This group R possesses all the required properties. Proposition (2.1) is therefore proved.

Theorem I. *Let W be a simply-connected manifold with* EULER *characteristic equal to one. If W admits transitively a connected compact transformation group, then W is a single point.*

Proof. By (2.1) there exists a connected, simply-connected compact semi-simple LIE group R acting on W transitively and almost effectively. Therefore, W can be regarded as a coset space R/L where L is a closed subgroup of R. From the simply-connectedness of both R and W, it follows that L is connected [1, § 31].

Since the EULER characteristic $\chi(W) = \chi(R/L) = 1 \neq 0$, R and L have the same rank l [4]. Hence a maximal toral subgroup T of L is, at the same time, a maximal toral subgroup of R. Let E^l be the tangent space of T. We have two finite linear groups $\Phi'(L)$, $\Phi'(R)$ of transformations of the space E^l. From the definition of the group Φ' and the fact $L \subset R$, it follows at once

$$\Phi'(L) \supset \Phi'(R). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

Concerning the EULER characteristic $\chi(R/L)$ and the orders $o(L)$, $o(R)$ of the finite group $\Phi'(L)$, $\Phi'(R)$, we have the formula [8, (1.1)]

$$\chi(R/L) = o(R)/o(L).$$

Our assumption $\chi(W) = \chi(R/L) = 1$ then implies that $o(R) = o(L)$, and then (2.2) tells us that

$$\Phi'(R) = \Phi'(L). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

Now we have two connected compact LIE groups L and R. Either of them has its own singular hyperplanes in the same euclidean space E^l. From the fact that $L \subset R$ we can see immediately that a singular element of L is also a singular element of R. Thus singular hyperplanes of L are also singular hyperplanes of R. Let us denote, respectively, by

$$\pi_1, \pi_2, \ldots, \pi_m$$

and

$$\pi_1, \pi_2, \ldots, \pi_m, \theta_1, \theta_2, \ldots, \theta_h,$$

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[2] A group is said to have no centre if it possesses no other centre than the identity.

the singular hyperplanes of L and R which pass through the origin. Then from (1.1), it follows that

$$r' = 2m + l, \quad r = 2(m + h) + l \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

where $r' = \dim. L$, $r = \dim. R$.

We are going to show $h = 0$. Suppose that $h \neq 0$. Then there exists a singular hyperplane θ_1 of R which is not a singular hyperplane of L and which passes through the origin. Choose a point P on θ_1 such that it does not lie on any of the hyperplanes $\pi_1, \pi_2, \ldots, \pi_m$. Since R is *semi-simple*, $\Phi'(R)$ contains the reflection φ' about the singular hyperplane θ_1 [7, p. 364]. On account of (2.3), φ' also belongs to $\Phi'(L)$. This φ' evidently differs from the identity transformation, and moreover it leaves the point P invariant. Proposition (1.2) then assures the existence of a singular hyperplane of L passing through both P and the origin. However, by the choice of P, none of the π's passes through P. This leads to a contradiction. Hence $h = 0$ and (2.4) implies $r = r'$. In other words, R and L have the same dimension. As R is connected, L and R coincide. Thus $W = R/L$ consists of only one point. This proves Theorem I.

3. Our main theorem can be stated as follows:

Theorem II. *Let W be a simply-connected manifold with* EULER *characteristic equal to two. If W admits, effectively and transitively, a compact connected group R, then W is a sphere of even dimension, and R is either the orthogonal group or the exceptional simple* LIE *group G_2 of* CARTAN*'s class G and rank 2. G_2 presents only when W is six-dimensional.*

In order to prove this theorem, we shall first establish a series of lemmas.

Lemma 1. *Let W be a simply-connected manifold with* EULER *characteristic equal to a prime number p. Then the connected compact group R which can possibly act on W transitively and effectively must be a simple* LIE *group.*

Proof. Let L denote the isotropic subgroup of R. Then W can be regarded as the coset space R/L. Since $\chi(W) = p \neq 0$, R has no centre [8, (2.1)]. If R is not simple, then R can be expressed as the direct product $R' \times R''$ of two connected compact semi-simple LIE groups none of which consists of only one element [8, (2.2)]. Then W is homeomorphic to the topological product

$$(R'/L') \times (R''/L'')$$

of two coset spaces where $L' = R' \cap L$, $L'' = R'' \cap L$ [8, (2.3)]. From the well-known KÜNNETH's formula, we have

$$p = \chi(W) = \chi(R'/L') \cdot \chi(R''/L'').$$

However, p is a prime number so that one of the factors in the right hand side of the above equality must be unity. We can assume that $\chi(R'/L') = 1$. Since W is simply-connected, R'/L' must be simply-connected as well.

Moreover, R'/L' admits transitively the compact LIE group R'. From Theorem I, R'/L' is a single point. Hence $R' = L'$.

Now we return to our original homogeneous space $W = R/L$. Since R' is an invariant subgroup of R and $R' = L' \subset L$, it follows that each element of R' leaves unaltered every point of W. R' has more than one elements. This contradicts our assumption that R is effective on W. Hence R is simple.

Lemma 2. *Let R_1 and R_2 be two locally isomorphic connected compact* LIE *groups. Then $\Phi(R_1) \approx \Phi(R_2)$.*

Proof. If R_1 and R_2 are semi-simple, this lemma is well-known. In the general case, we know that there exists a connected compact LIE group R such that [6, Theorem 87]

$$R_1 = R/N_1, \qquad R_2 = R/N_2$$

where N_1 and N_2 are discrete subgroups of R belonging to the centre. Hence we have two natural homomorphic onto-mappings

$$f_1 : R \to R_1, \quad f_2 : R \to R_2$$

with kernels N_1 and N_2 respectively. Let T be a maximal total subgroup of R. N_1 and N_2 being contained in the centre of R, are contained in T. From HOPF's result [3, 1.6], it follows that

$$\Phi(R) \approx \Phi(R_1), \ \Phi(R) \approx \Phi(R_2),$$

and hence $\Phi(R_1) \approx \Phi(R_2)$.

Lemma 3. *Let L be a connected compact* LIE *group locally isomorphic to the direct product*

$$L_1 \times L_2 \times \ldots \times L_s$$

of connected compact LIE *groups L_j $(j = 1, 2, \ldots, s)$. Then*

$$o(L) = o(L_1) \cdot o(L_2) \ldots o(L_s).$$

Proof. This is a direct consequence of Lemma 2 and [8, (14.1)].

4. In this section and the next, we shall study some properties of the CARTAN's exceptional groups.

Lemma 4. *The* CARTAN's *exceptional group F_4, E_6, E_7, E_8 cannot act transitively on a simply-connected manifold with* EULER *characteristic equal to two.*

Proof. Let us first consider the case E_8. Suppose W to be a simply-connected manifold with EULER characteristic equal to two. If there exists an E_8 acting on W transitively, then the universal covering group $\widetilde{E}_8$ of E_8 acts on W in the natural manner. This group $\widetilde{E}_8$ is simply-connected and is also of the class E_8. Let L be the isotropic of subgroup $\widetilde{E}_8$. We can

regard W as the coset space $\widetilde{E}_8/L$. From the simply-connectedness of both W and $\widetilde{E}_8$, it follows that L is connected [1, § 31]. Therefore [8, (1.1)]

$$2 = \chi(W) = \chi(\widetilde{E}_8/L) = o(\widetilde{E}_8)/o(L).$$

The table (1.3) tells us that $o(\widetilde{E}_8) = 10! \cdot 3 \cdot 2^6$. Hence we have

$$o(L) = 10! \cdot 3 \cdot 2^5. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.1)$$

The isotropic subgroup L, being closed in $\widetilde{E}_8$, form a compact LIE group. Hence it is locally isomorphic with a direct product of the form

$$L_1 \times L_2 \times \ldots \times L_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

where L_j is either a toral group or a compact simple LIE group. Let us denote by l_j the rank of L_j. Then

$$l_1 + l_2 + \ldots + l_s = \text{rank of } L.$$

Since $\chi(\widetilde{E}_8/L) \neq 0$, L has the same rank 8 as the group $\widetilde{E}_8$, and hence

$$l_1 + l_2 + \ldots + l_s = 8. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.3)$$

Furthermore, we have from Lemma 3 that $o(L) = o(L_1) \cdot o(L_2) \ldots o(L_s)$. Equality (4.1) then implies

$$o(L_1) \cdot o(L_2) \ldots o(L_s) = 10! 3 \cdot 2^5. \quad . \quad . \quad . \quad . \quad . \quad (4.4)$$

Evidently L_j cannot be an E_8, and from (4.3) it follows that $l_j \leq 8$. Moreover, one of the $o(L_j)$ must be divisible by 7. For definiteness, let it be $o(L_1)$. Table (1.3) then tells us that L_1 is one of the following groups

$$A_8, A_7, A_6, B_8, B_7, C_8, C_7, D_8, D_7, E_7.$$

Hence $o(L_1)$ is not divisible by 25. One of the factors $o(L_2), \ldots, o(L_s)$ must be divisible by 5. However, $l_j \leq 8 - l_1 \leq 2$ $(j = 2, 3, \ldots, s)$. It follows from (1.3) that this is impossible.

From the above discussions, we know that an E_8 cannot act transitively on W. Thus the lemma is proved for the case E_8. The other cases can be treated similarly. Q. E. D.

Lemma 5. *Let W be a simply-connected space with* EULER *characteristic equal to two. If it admits transitively the exceptional group G_2, then the isotropic subgroup must be an A_2.*

Proof. Let L denote the isotropic subgroup. By similar reasoning as above, we know that L is of rank 2 and $o(L) = 6$. (1.3) and Lemma 3 tell us that L is an A_2.

5. In the preceding section, we know that there is possibly a simply-connected homogeneous space of G_2 with EULER characteristic equal to two. Now we shall show that this space is the sphere of six dimension.

Lemma 6. *Up to an automorphism of G_2, there is one and only one subgroup L of G_2 such that L is an A_2.*

Proof. Noting the fact that G_2 is simply-connected [7, p. 378], we can prove this lemma by reasoning analogous to, though far simpler than, that the author used in [8, Part II]. The details are omitted.

Lemma 7. *The six-dimensional sphere S^6 is a homogeneous space of the group G_2, and is the only simply-connected homogeneous space of G_2 which has* EULER *characteristic equal to two.*

Proof. The uniqueness follows directly from Lemma 6 and Lemma 7. We need only prove the first part of our lemma. It is well-known that G_2 can be embedded in the group Γ_6 of all proper orthogonal matrices with 7 rows and columns. Therefore, we can assume $G_2 \subset \Gamma_6$.

Let S^6 be the unit sphere in the seven-dimensional euclidean space. Γ_6 acts transitively and effectively on S^6. G_2, being a subgroup of Γ_6, acts on S^6 in the natural manner. We shall show that G_2 is transitive on S^6. For this purpose, let us consider the orbits of G_2 [10, p. 194]. Let m be the maximum of the dimension of all the orbits. There is a point x of S^6 such that the orbit $G_2(x)$ is m-dimensional. Since each orbit is connected and G_2 acts effectively on S^6, we have

$$0 < m \leqq 6 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.1)$$

The group G_2 acts transitively on $G_2(x)$ so that $G_2(x)$ can be regarded as a coset space G_2/L where L is a closed subgroup of G_2. Since G_2 is a simple LIE group without centre [7, p. 378], it has no proper invariant subgroup. Furthermore,

$$\text{dim. } G_2 - \text{dim. } L = \text{dim. } G_2(x) = m > 0.$$

Thus $L \neq G_2$ and thus G_2 acts effectively on $G_2(x)$ [1, § 18]. It follows then [10, p. 202]

$$14 = \text{dim. } G_2 \leqq m\,(m+1)/2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.2)$$

Combining (5.1) and (5.2), we know that m is either equal to 5 or equal to 6.

Suppose $m = 5$. Then [5, p. 465] all orbits of G_2 are 5-dimensional except for two orbits of lower dimension. Let these two exceptional orbits be $G_2(y_1)$ and $G_2(y_2)$ $(y_1, y_2 \varepsilon S^6)$. Then $G_2(y_i)$ must be of zero dimension, for otherwise by using similar reasoning as above we can show that its dimension is either 5 or 6. Hence $G_2(y_i) = y_i$. In other words, y_i $(i = 1, 2)$ is a fixed point of G_2. All the matrices of Γ_6 leaving y_1 unaltered form an orthogonal group Γ_5 of order 6. Hence

$$G_2 \supset \Gamma_5. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.3)$$

However, Γ_5 cannot have any proper subgroup of dimension greater than ten [5, p. 463]. Hence (5.3) is absurd. The integer m cannot be equal to 5, and therefore $m = 6$. It follows then that $G_2(x) = S^6$. In other words, S^6 is really a homogeneous space of G_2. This completes the proof.

6. In a recent paper [8], the author has determined all the spaces which have non-vanishing EULER characteristic and admit transitively a classical

group (i.e., simple group of the four main classes). According as the group is an A_k, B_k, C_k, or D_k, the space is called an elementary χ_{A^-}, χ_{B^-}, χ_{C^-} or χ_{D^-}space respectively, and we denote it by W_A, W_B, W_C or W_D. To each $W_A (W_B, W_C, W_D)$ is associated a set $\Theta_A (\Theta_B, \Theta_C, \Theta_D)$ of integers, called category [8, § 9], given as follows:

$$\Theta_A = \{e; a_1, a_2, \ldots, a_n\}, \quad \ldots \quad (6.1A)$$

$$\Theta_B = \{e; a_1, a_2, \ldots, a_n; d_1, d_2, \ldots, d_{m'}; b\}, \quad \ldots \quad (6.1B)$$

$$\Theta_C = \{e; a_1, a_2, \ldots, a_u; c_1, c_2, \ldots, c_m\}, \quad \ldots \quad (6.1C)$$

$$\Theta_D = \{e; a_1, a_2, \ldots, a_n; d_1, d_2, \ldots, d_m\}. \quad \ldots \quad (6.1D)$$

These categories $\Theta_A, \Theta_B, \Theta_C, \Theta_D$ satisfy, respectively, the following conditions

$$e \geqq n-1, \ a_\alpha \geqq 2, \ e + \Sigma a_\alpha = l, \quad \ldots \quad (6.2A)$$

$$e \geqq n, \ a_\alpha \geqq 2. \ d_\beta \geqq 2, \ b \geqq 0, \ e-n + \Sigma a_\alpha + \Sigma d_\beta = l, \quad (6.2B)$$

$$e \geqq n, \ a_\alpha \geqq 2, \ c_\beta \geqq 1, \ e-n + \Sigma a_\alpha + \Sigma c_\beta = l, \quad \ldots \quad (6.2C)$$

$$e \geqq n, \ a_\alpha \geqq 2, \ d_\beta \geqq 2, \ e-n + \Sigma a_\alpha + \Sigma d_\beta = l. \quad \ldots \quad (6.2D)$$

where l denotes the rank of the group which the space admits. It has been shown that [8, § 13, Corollary 6]

(6.3) *Two simply-connected elementary χ-spaces with the same category are homeomorphic.*

Let W_A, W_B, W_C and W_D be *simply-connected* elementary χ-spaces of category $\Theta_A, \Theta_B, \Theta_C$ and Θ_D respectively. Concerning their EULER characteristic χ, we have the formulae

$$\left.\begin{aligned}
\chi(W_A) &= \frac{(l+1)!}{a_1!\, a_2! \ldots a_n!} \\
\chi(W_B) &= \frac{2^{(l+m'-b-d_1-d_2-\ldots-d_{m'})}\, l!}{a_1! \ldots a_n!\, d_1! \ldots d_{m'}!\, b!} \\
\chi(W_C) &= \frac{2^{(-c_1-c_2-\ldots-c_m)}\, l!}{a_1! \ldots a_n!\, c_1! \ldots c_m!} \\
\chi(W_D) &= \frac{2^{(l+m-d_1-d_2-\ldots-d_m-1)}\, l!}{a_1! \ldots a_n!\, d_1! \ldots d_m!}
\end{aligned}\right\} \quad \ldots \quad (6.4)$$

From (6.2) and (6.4), it is easy to single out those spaces whose EULER characteristic is equal to two. In fact, we have

Lemma 8. *Let W be a simply-connected space which has* EULER *characteristic equal to two and admits transitively and effectively a classical group R. Then W is homeomorphic with a sphere of even dimension, and R is the orthogonal group.*

Proof. Since R is a classical group and $\chi(W) \neq 0$, W is an elementary χ_{A^-}, χ_{B^-}, χ_{C^-} or χ_{D^-}space. Bearing in mind the simply-connectedness of W, we can easily see from (6.2) and (6.4) that R must be one of the following groups

$$A_1, C_1, C_2, B^l \ (l = 1, 2, \ldots).$$

However, it is well-known that $A_1 = C_1 = B_1$, $C_2 = B_2$ so that R is an B_l and W is an elementary χ_B-space. By hypothesis, R acts on W effectively. Hence R has no centre [8, (2.1)] and hence R is the group of all proper orthogonal matrices with $2l + 1$ rows and columns. Moreover, (6.4) tells us that the category Θ_B of the space $W = B_l/L$ is specified as follows

$$e = b = n = 0, \ m' = 1, \ d_1 = l. \quad . \quad . \quad . \quad . \quad . \quad (6.5)$$

On the other hand, the $2l$-dimensional sphere S^{2l} is an elementary χ_B-space B_l/D_l. From the very definition of category, it follows that the category of S^{2l} is also given by (6.5). On account of (6.3), W and S^{2l} are homeomorphic. The lemma is thus proved.

7. Proof of Theorem II. Let W be a simply-connected manifold admitting effectively and transitively a connected compact group R, and having EULER characteristic equal to two. From Lemma 1, we know that R is a connected compact simple LIE group. Lemma 4 tells us that R cannot be of the classes F_4, E_6, E_7, E_8. Hence R is either the exceptional group G_2 or a classical group.

If R is the group G_2, it follows from Lemma 7 that W is the six-dimensional sphere. In the other alternative, Lemma 8 tells us that W is a sphere of even dimension and R the orthogonal group. Theorem II is therefore proved.

On account of Lemmas 5 and 7, we know that S^6 is homeomorphic with the coset space G_2/A_2. From this fact and the well-known homotopy sequence, it follows immediately

Let $\pi_n(G_2)$ denote the n^{th} homotopy group of the exceptional group G_2. Then $\pi_3(G_2)$ is free cyclic and

$$\pi_1(G_2) = \pi_2(G_2) = \pi_4(G_2) = 0.$$

Furthermore, by using similar method as in the proof of Theorem II we can prove the following

Theorem III [3]. *Let W be a simply-connected manifold with* EULER *characteristic equal to a prime number $p > 2$. If it admits transitively a compact group of transformations. Then W is either a complex projective space of $2(p-1)$ dimension, or a quaternion projective space of $4(p-1)$*

[3]) From a recent personal correspondence, the author learnt that some of these result are known to A. BOREL.

dimension or a 16-dimensional closed orientable manifold with POINCARE *polynomial*

$$1 + t^8 + t^{16}.$$

Academia Sinica.

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Mathematics. — *On the small vibrations of non-holonomic systems.* By O. BOTTEMA. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of September 24, 1949.)

The small vibratory motions of a conservative non-holonomic system about a position of equilibrium have been considered by WHITTAKER in his *Analytical Dynamics* [1]). His conclusions are that here "the difference between holonomic and non-holonomic systems is unimportant" and that the vibratory motion of a given non-holonomic system with n independant coordinates and m non-integrable kinematical relations is the same as that of a certain holonomic system with $n-m$ degrees of freedom.

These conclusions appear to be incorrect.

For the sake of simplicity we first consider a system with *three* coordinates x_1, x_2, x_3, kinetic energy

$$T = \tfrac{1}{2}(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and potential energy $V(x_1, x_2, x_3)$. Suppose that there is *one* kinematical relation, integrable or non-integrable:

$$A_1 \dot{x}_1 + A_2 \dot{x}_2 + A_3 \dot{x}_3 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where A_i $(i = 1, 2, 3)$ are functions of x_1, x_2 and x_3.

Suppose further that $x_1 = x_2 = x_3 = 0$ is a position of equilibrium. This means: if for $t = 0$ we have $x_1 = x_2 = x_3 = \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$ then the system is permanently at rest. What can be said then about $V(x_1 x_2 x_3)$? We write down the Lagrangian equations

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_i} - \frac{\partial T}{\partial x_i} = -\frac{\partial V}{\partial x_i} + \lambda A_i$$

or

$$\ddot{x}_i = -\frac{\partial V}{\partial x_i} + \lambda A_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where λ is a function of t only. Substitution of $x_i = 0$ gives

$$\frac{\partial V(0,0,0)}{\partial x_i} = \lambda_0 A_i(0,0,0). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence for $x_1 = x_2 = x_3 = 0$ the $\partial V/\partial x_i$ are proportional to the coefficients A_i (which is of course obvious in view of the dynamical meaning of

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[1]) E. T. WHITTAKER, A treatise on the Analytical Dynamics of particles and rigid bodies, fourth revised edition (1936); first american printing, New York (1944), pg. 221—226.

$\partial V/\partial x_i$ and the geometrical meaning of (2)). So for $x_i = 0$ it is not necessary that $\partial V/\partial x_i = 0$, as WHITTAKER supposes.

We may take $\lambda_0 = 1$ without loss of generality in view of the fact that the left member of the kinematical relation (2) can be multiplied with the constant factor λ_0.

Now we consider the motion of the system in the neighbourhood of $x_i = 0$, so that the x_i are small.

We have then, in view of (4):

$$V = a_1 x_1 + a_2 x_2 + a_3 x_3 + \tfrac{1}{2} \Sigma b_{ij} x_i x_j \qquad (b_{ij} = b_{ji}) \quad . \quad . \quad (5)$$

and the kinematical relation (2) is

$$\left.\begin{aligned} &(a_1 + c_{11} x_1 + c_{12} x_2 + c_{13} x_3)\, \dot{x}_1 + \\ &+ (a_2 + c_{21} x_1 + c_{22} x_2 + c_{23} x_3)\, \dot{x}_2 + (a_3 + c_{31} x_1 + c_{32} x_2 + c_{33} x_3)\, \dot{x}_3 = 0 \end{aligned}\right\} \quad (6)$$

so that the equations of motion are as follows

$$\left.\begin{aligned} &\ddot{x}_1 + a_1 + b_{11} x_1 + b_{12} x_2 + b_{13} x_3 - \lambda (a_1 + c_{11} x_1 + c_{12} x_2 + c_{13} x_3) = 0 \\ &\ddot{x}_2 + a_2 + b_{21} x_1 + b_{22} x_2 + b_{23} x_3 - \lambda (a_2 + c_{21} x_1 + c_{22} x_2 + c_{23} x_3) = 0 \\ &\ddot{x}_3 + a_3 + b_{31} x_1 + b_{32} x_2 + b_{33} x_3 - \lambda (a_3 + c_{31} x_1 + c_{32} x_2 + c_{33} x_3) = 0 \end{aligned}\right\} . \quad (7)$$

If we put $\lambda = 1 - \varepsilon$, then ε is small and of the same order as x_i. Taking into account in (6) and (7) linear terms only, we have

$$a_1 \dot{x}_1 + a_2 \dot{x}_2 + a_3 \dot{x}_3 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

and

$$\ddot{x}_1 + b_{11} x_1 + b_{12} x_2 + b_{13} x_3 + \varepsilon a_1 - (c_{11} x_1 + c_{12} x_2 + c_{13} x_3) = 0 \quad (7a)$$

(6a) and (7a) form a system of four lineair homogeneous differential equations for the four unknown functions x_1, x_2, x_3, ε. Substituting $x_i = C_i e^{pt}$, $\varepsilon = C_0 e^{pt}$ and eliminating the constants C we find an equation for p. One of the roots is $p = 0$ and the others are the roots of

$$D \equiv \begin{vmatrix} b_{11} - c_{11} + p^2 & b_{12} - c_{12} & b_{13} - c_{13} & a_1 \\ b_{21} - c_{21} & b_{22} - c_{22} + p^2 & b_{23} - c_{23} & a_2 \\ b_{31} - c_{31} & b_{32} - c_{32} & b_{33} - c_{33} + p^2 & a_3 \\ a_1 & a_2 & a_3 & 0 \end{vmatrix} = 0, \quad . \quad (8)$$

which is of the second degree in p^2, so that we have in all five roots p. The root $p = 0$ is of course an unpleasant one and asks for a further investigation of the system if one inquires about the stability of equilibrium.

As for the other roots, the determinant (8) is in general *not symmetrical* because c_{ij} is not necessarily equal to c_{ji} and here we have a difference with the holonomic case. In order to prove that the difference is essential we give the following example. Let $a_1 = a_2 = 0$, $a_3 = a \neq 0$, $C_{ij} = 0$ with the exception of $c_{21} = c \neq 0$, so that we have the system

$$V = a x_3 + \tfrac{1}{2} \Sigma b_{ij} x_i x_j$$

and the relation

$$c x_1 \dot{x}_2 + a \dot{x}_3 = 0$$

which is non-holonomic. The equation (8) is now

$$p^4 + (b_{11} + b_{22})\, p^2 + b_{11}\, b_{22} - b_{12}\,(b_{12} - c) = 0$$

and its discriminant

$$d = (b_{11} - b_{22})^2 + 4\, b_{12}^2 - 4\, c b_{12}$$

is obviously not always positive. We know however that for a holonomic system, stable or unstable, the values of p^2 are always real.

If the system is holonomic the relation (6) is integrable, hence $c_{ij} = c_{ij}$ and (8) is symmetrical.

We have to consider the case that (6) is not integrable as it stands but can be made so by as integrating factor. If we multiply the left member by $(1 + q_1 x_1 + q_2 x_2 + q_3 x_3 + \ldots)$ and take into account only linear terms, we get

$$c'_{ij} = c_{ij} + a_i\, q_j$$

and it is easily seen that D' is obtained from D by multiplying the last column by q_1 and adding it to the j-th column. Hence we have: for holonomic systems D is symmetrical or can be made so by elementary transformations; for non-holonomic systems D is essentially unsymmetrical.

For the general case of n coordinates x_i and m kinematical relations, where $x_i = 0$ is a position of equilibrium, we have

$$T = \tfrac{1}{2} \Sigma a_{ij}\, \dot{x}_i\, \dot{x}_j, \quad V = \Sigma a_i\, x_i + \Sigma b_{ij}\, x_i\, x_j \qquad (a_{ij} = a_{ji},\ b_{ij} = b_{ji})$$

$$\sum_{ij} (a_{ik} + c_{ijk}\, x_j)\, \dot{x}_i = 0 \qquad (k = 1, 2, \ldots m).$$

The equation for the frequencies has m roots $p = 0$ and the other $2(n-m)$ roots are those of

$$D \equiv \begin{vmatrix} b_{11} - \Sigma c_{11k} + a_{11} p^2 & b_{12} - \Sigma c_{12k} + a_{12} p^2 \ldots b_{1n} - \Sigma c_{1nk} + a_{1n} p^2 & a_{11}\ a_{12} \ldots a_{1m} \\ b_{21} - \Sigma c_{21k} + a_{21} p^2 & b_{22} - \Sigma c_{22k} + a_{22} p^2 \ldots b_{2n} - \Sigma c_{2nk} + a_{2n} p^2 & a_{21}\ a_{22} \ldots a_{2m} \\ \ldots & \ldots & \ldots \\ b_{n1} - \Sigma c_{n1k} + a_{n1} p^2 & b_{n2} - \Sigma c_{n2k} + a_{n2} p^2 \ldots b_{nn} - \Sigma c_{nnk} + a_{nn} p^2 & a_{n1}\ a_{n2} \ldots a_{nm} \\ a_{11} & a_{21} \qquad\qquad a_{n1} & 0\ \ 0 \ldots 0 \\ a_{12} & a_{22} \qquad\qquad a_{n2} & 0\ \ 0 \ldots 0 \\ \ldots & \ldots & \ldots \\ a_{1m} & a_{2m} \qquad\qquad a_{nm} & 0\ \ 0 \ldots 0 \end{vmatrix} = 0.$$

For holonomic systems D is symmetrical or can be made so by elementary transformations. This form of the determinant, consisting of an ordinary symmetrical p^2-determinant of the n-th order, to which are added m rows and m columns of constants is given by ROUTH [2]), who, however, takes the relations in integrated form so that the m roots $p = 0$ do not occur.

For non-holonomic systems D is essentially non-symmetrical.

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[2]) ROUTH, The advanced part of a treatise on the dynamics of a system of rigid bodies, Chapter II.

Mathematics. — *On the uniform distribution modulo 1 of sequences* $(f(n,\theta))$. By P. ERDÖS and J. F. KOKSMA. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of June 25, 1949.)

I. *Introduction.* In a former paper [1]) we treated lacunary sequences. Now, using an other method, we consider general sequences. For notation and definitions, see [1]). We prove

Theorem 1. *Let* $f(1,\theta)$, $f(2,\theta)$, ... *be a sequence of real numbers, defined for each value of* θ *of the segment* $\alpha \leqq \theta \leqq \beta$, *such that* $f(n,\theta)$ *for* $n = 1, 2, \ldots$ *as a function of* θ, *has a continuous derivative* f'_θ *and such that the expression*

$$f'_\theta(n_1,\theta) - f'_\theta(n_2,\theta)$$

for each couple of positive integers $n_1 \neq n_2$ *is either a non-decreasing or a non-increasing function of* θ *on* $\alpha \leqq \theta \leqq \beta$, *the absolute value of which is* $\geqq \delta$, *where* δ *denotes a positive number which does not depend on* n_1, n_2, *or* θ. *Then for almost all* θ *the discrepancy* $D(N,\theta)$ *of the sequence satisfies the inequality*

$$ND(N,\theta) = O(N^{\frac{1}{2}} \log^{\frac{3}{2}+\varepsilon} N) \qquad (\varepsilon > 0). \quad . \quad . \quad . \quad . \quad (1)$$

Theorem 1 is a special case of the more general

Theorem 2. *Let* $f(n,\theta)$ *for* $n = 1, 2, \ldots$ *denote a real continuous function of* θ *on* $\alpha \leqq \theta \leqq \beta$ *and let*

$$\varphi(n_1, n_2, \theta) = f(n_1,\theta) - f(n_2,\theta) \text{ for } n_1 \neq n_2$$

have a continuous derivative φ'_θ *which is* $\neq 0$ *and either non-decreasing or non-increasing on* $\alpha \leqq \theta \leqq \beta$. *Put*

$$A(M,N) = \frac{1}{N^2} \sum_{n_1=M+2}^{M+N} \sum_{n_2=M+1}^{n_1-1} \operatorname{Max}\left(\frac{1}{|\varphi'_\theta(n_1,n_2,\alpha)|}, \frac{1}{|\varphi'_\theta(n_1,n_2,\beta)|} \right)$$

and assume that for some constant $\gamma \geqq 1$

$$NA(M,N) \leqq K_0 \log^\gamma N \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2$$

for all couples of positive integers M, N *where* K_0 *is a positive constant.*

Then for almost all numbers θ *in* $\alpha \leqq \theta \leqq \beta$ *the discrepancy* $D(N,\theta)$ *of the sequence* $f(1,\theta)$, $f(2,\theta)$, ... *satisfies the inequality*

$$ND(N) = O(N^{\frac{1}{2}} \log^{\frac{\gamma+4+\varepsilon}{2}} N) \qquad (\varepsilon > 0).$$

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[1]) P. ERDÖS and J. F. KOKSMA, On the uniform distribution modulo 1 of lacunary sequences. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **52**, 264—273 (1949). (= Indag. Math. **11**, 79—88 (1949).)

Remarks. 1. It is clear that the functions $f(n, \theta)$ of Theorem 1 satisfy the assumptions of Theorem 2. For if one ranges the N numbers

$$f(M+1, \theta),\ f(M+2, \theta), \ldots,\ f(M+N, \theta)$$

in order of magnitude, these numbers at each step increase with at least the amount δ and we find

$$\sum_{n_2=M+1}^{n_1-1} \frac{1}{|f'_\theta(n_1, \theta) - f'_\theta(n_2, \theta)|} < 2 \sum_{\mu=1}^{N} \frac{1}{\mu\delta} < \frac{2}{\delta} \log 3N,$$

hence

$$NA(M, N) \leqq \frac{2}{\delta} \log 3N = O(\log^\gamma N) \text{ for } \gamma = 1.$$

2. As Mr J. W. S. CASSELS has shown us, he also proved Theorem 1. His very interesting method is different from ours. The proofs are completely independent from each other.

II. *Some lemma's.*

Lemma 1. *Let $f(n, \theta)$ for $n = 1, 2, \ldots$ denote a real continuous function of θ on $\alpha \leqq \theta \leqq \beta$ and let*

$$\Phi(n_1, n_2, \theta) = f(n_1, \theta) - f(n_2, \theta) \text{ for } n_1 \neq n_2$$

have a continuous derivative Φ'_θ which is $\neq 0$ and either non-decreasing or non-increasing on $\alpha \leqq \theta \leqq \beta$. Finally put

$$A_N = \frac{1}{N^2} \sum_{n_1=2}^{N} \sum_{n_2=1}^{n_1-1} \operatorname{Max}\left(\frac{1}{|\Phi'_\theta(n_1, n_2, \alpha)|}, \frac{1}{|\Phi'_\theta(n_1, n_2, \beta)|}\right).$$

Then we have for $N \geqq 2$, $h > 0$ (h not depending on n and θ)

$$\int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right|^2 d\theta \leqq (\beta - \alpha) N + \frac{A_N}{h} N^2.$$

The proof of this lemma has been given by KOKSMA [3].

Lemma 2. *If $u_1, u_2, \ldots$ is a real sequence and if $D(N)$ denotes its discrepancy then for each integer $m \geqq 1$, we have*

$$ND(N) \leqq K\left(\frac{N}{m+1} + \sum_{h=1}^{m} \frac{1}{h} \left| \sum_{n=1}^{N} e^{2\pi i h u_n} \right|\right),$$

where K denotes a numerical constant.

This lemma is an improvement proved by ERDÖS–TURÁN [4] of the one-dimensional case of a theorem of VAN DER CORPUT–KOKSMA [5].

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[2] For litt. see [1] and also [5].

[3] J. F. KOKSMA, Ein mengentheoretischer Satz über die Gleichverteilung modulo Eins. Comp. Math. **2**, 250—258 (1935).

[4] P. ERDÖS and P. TURÁN, On a problem in the theory of uniform distribution. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **51**, 1146—1154, 1262—1269 (1948). (= Indag. Math. **10**, 370—378, 406—413 (1948).)

[5] See J. F. KOKSMA, Diophantische Approximationen, Erg. d. Math. IV, 4 (1936), Kap. IX.

Lemma 3. *If $f(n,\theta)$ denotes the function of Lemma 1, and if $D(N,\theta)$ denotes the discrepancy of the sequence $f(1,\theta), f(2,\theta), \ldots$, then*

$$\int_\alpha^\beta N^2 D^2(N,\theta)\, d\theta \leqq K_1 (N \log^2 N + A_N N^2 \log N).$$

where K_1 depends on $\beta - \alpha$ only.

Proof. Putting $m = [\sqrt{N}]$, we have by Lemma 2

$$N^2 D^2(N,\theta) \leqq K^2 \left(N + 2\sqrt{N} \sum_{h=1}^{[\sqrt{N}]} \frac{1}{h} \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right| \right) +$$

$$+ K^2 \sum_{h=1}^{[\sqrt{N}]} \sum_{k=1}^{[\sqrt{N}]} \frac{1}{hk} \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right| \cdot \left| \sum_{n=1}^{N} e^{2\pi i k f(n,\theta)} \right|.$$

Hence

$$\int_\alpha^\beta N^2 D^2(N,\theta)\, d\theta \leqq K^2 \left(N(\beta-\alpha) + 2\sqrt{N} \sum_{h=1}^{[\sqrt{N}]} \frac{1}{h} \int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right| d\theta \right) +$$

$$+ K^2 \sum_{h=1}^{[\sqrt{N}]} \sum_{k=1}^{[\sqrt{N}]} \frac{1}{hk} \int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right| \left| \sum_{n=1}^{N} e^{2\pi i k f(n,\theta)} \right| d\theta$$

and by the CAUCHY–SCHWARZ inequality for integrals

$$\leqq K^2 \left(N(\beta-\alpha) + 2\sqrt{N} \sum_{h=1}^{[\sqrt{N}]} \frac{1}{h} \left\{ \int_\alpha^\beta 1^2\, d\theta \cdot \int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right|^2 d\theta \right\}^{\frac{1}{2}} \right) +$$

$$+ K^2 \sum_{h=1}^{[\sqrt{N}]} \sum_{k=1}^{[\sqrt{N}]} \frac{1}{hk} \left\{ \int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i h f(n,\theta)} \right|^2 d\theta \cdot \int_\alpha^\beta \left| \sum_{n=1}^{N} e^{2\pi i k f(n,\theta)} \right|^2 d\theta \right\}^{\frac{1}{2}}$$

$$\leqq K^2 \left(N(\beta-\alpha) + 2\sqrt{N} \sum_{h=1}^{[\sqrt{N}]} \frac{1}{h} \left\{ (\beta-\alpha)^2 N + \frac{\beta-\alpha}{h} A_N \cdot N^2 \right\}^{\frac{1}{2}} \right) +$$

$$+ \sum_{h=1}^{[\sqrt{N}]} \sum_{k=1}^{[\sqrt{N}]} \frac{1}{hk} \left\{ (\beta-\alpha) N + \frac{1}{h} A_N \cdot N^2 \right\}^{\frac{1}{2}} \left\{ (\beta-\alpha) N + \frac{1}{k} A_N \cdot N^2 \right\}^{\frac{1}{2}}$$

by Lemma 1. Hence by the CAUCHY–SCHWARZ-inequality for sums

$$\int_\alpha^\beta N^2 D^2(N,\theta)\, d\theta \leqq K^2 \left(N(\beta-\alpha) + 2\sqrt{N} \left\{ \sum_{h=1}^{N} \frac{\beta-\alpha}{h} \sqrt{N} + \sum_{h=1}^{N} \frac{\sqrt{\beta-\alpha}}{h\sqrt{h}} \sqrt{A_N} \cdot N \right\} + \right.$$

$$\left. + \left\{ \sum_{h=1}^{N} \sum_{k=1}^{N} \frac{1}{hk} \left\{ (\beta-\alpha) N + \frac{1}{h} A_N \cdot N^2 \right\} \cdot \sum_{h=1}^{N} \sum_{k=1}^{N} \frac{1}{hk} \left\{ (\beta-\alpha) N + \frac{1}{k} A_N \cdot N^2 \right\} \right\}^{\frac{1}{2}} \right)$$

$$\leqq K_2 (N + N \log N + \sqrt{A_N}\, N^{\frac{1}{2}} + N \log^2 N + A_N N^2 \log N),$$

where K_2 only depends on K and $\beta - \alpha$.

Now if

$$\sqrt{A_N}\, N^{\frac{1}{2}} > A_N N^2 \log N,$$

we should have

$$1 > \sqrt{A_N}\sqrt{N} \log N,$$

hence

$$A_N N^2 \log N < \sqrt{A_N}\, N^{\frac{3}{2}} < N.$$

Therefore

$$\int_\alpha^\beta N^2 D^2(N,\theta)\, d\theta \leqq K_1 (N \log^2 N + A_N N^2 \log N).$$

Q.e.d.

Lemma 4. *Let $F(M,N) = F(M,N,\theta)$ denote a function of θ on a segment $\alpha \leqq \theta \leqq \beta$ for each couple of positive integers M and N, such that*

$$|F(M,N)| \leqq |F(M,N_1)| + |F(M+N_1, N-N_1)| \quad . \quad . \quad . \quad (3)$$

for each triple M, N and $N_1 \leqq N$ and such that F belongs to the class L^2 over the segment. Let further

$$\int_\alpha^\beta |F(M,N,\theta)|^2\, d\theta \leqq K_3 N \log^\sigma N$$

$K_3 > 0$ and σ, being real constants. Then for almost all θ in $\alpha \leqq \theta \leqq \beta$ we have

$$|F(0,N,\theta)| = O(N^{\frac{1}{2}} \log^{\frac{\sigma+3+\varepsilon}{2}} N) \quad (\varepsilon > 0).$$

This lemma is a special case of a theorem of GÁL–KOKSMA, the proof of which will appear before long [6]).

III. We now prove Theorem 2.

Let M denote an arbitrary integer $\geqq 1$ and consider the functions

$$f(M+1,\theta),\ f(M+2,\theta), \ldots; \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

these functions satisfy the assumptions of Lemma 1 and the corresponding number A_N is exactly identical with the number $A(M,N)$ which we have defined in Theorem 2. Denoting the discrepancy of the sequence (4) by $D(M,N,\theta)$, we have by Lemma 3, applied to the sequence (4),

$$\int_\alpha^\beta N^2 D^2(M,N,\theta)\, d\theta \leqq K_1 (N \log^2 N + A(M,N) N^2 \log N) \leqq K_4 N \log^{1+\gamma} N$$

because of (2). Now it is easily seen from the definition of $D(N)$, that if we put

$$F(M,N,\theta) = N D(M,N,\theta);$$

the relation (3) is satisfied. Hence Theorem 2 follows immediately from Lemma 4 with $\sigma = 1 + \gamma$.

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[6]) Cf. I. S. GÁL et J. F. KOKSMA, Sur l'ordre de grandeur des fonctions sommables. C. R. Acad. d. Sc. Paris, 227, 1321—1323 (1948).

Mathematics. — *On the Geometry of a System of Partial Differential Equations of the Second Order.* By A. URBAN. (Communicated by Prof. J. A. SCHOUTEN.)

(Communicated at the meeting of September 24, 1949.)

1. The geometry of the system of partial differential equations of the second order

$$\partial^2_{\mu\lambda} z^h = H^h_{\mu\lambda}(\xi^\varkappa, z^j, \partial_\nu z^k); \quad \varkappa, \lambda, \mu, \nu = 1, \ldots, n; \quad h, i, j, k = 1, \ldots, m, \quad (1.1)$$

and similar systems of higher order with the unknown functions z^h and the independent variables $\xi^\varkappa$ was treated already many times. An extensive bibliography of the subject is found in E. BORTOLOTTI [3] [1]). The usual geometrical interpretation of the system (1.1) may be characterized as follows. If the invariance of the equations (1.1) for certain transformations in the variables $\xi^\varkappa$ or $\xi^\varkappa$, z^h is established and if from the $H^h_{\mu\lambda}$ the components of some linear connexion and (or) some other geometric objects in the space of the $\xi^\varkappa$ or $\xi^\varkappa$, z^h can be derived such that vice versa the $H^h_{\mu\lambda}$ can be derived from the components, the geometry in this space established by these objects gives the geometric interpretation of the given system of equations.

All this is very simple for $m = 1$ and for $H^1_{\mu\lambda}$ linear in $\partial_\lambda z$ and z

$$\partial^2_{\mu\lambda} z = \Gamma^\varkappa_{\mu\lambda} \partial_\varkappa z + \Gamma_{\mu\lambda} z; \quad \varkappa, \lambda, \mu = 1, \ldots, n. \quad . \quad . \quad . \quad (1.2)$$

For the transformations

$$\xi^{\varkappa'} = \xi^{\varkappa'}(\xi^\varkappa); \quad \varkappa = 1, \ldots, n; \quad \varkappa' = 1', \ldots, n'; \quad . \quad . \quad . \quad (1.3)$$

the functions $\Gamma^\varkappa_{\mu\lambda}$ resp. $\Gamma_{\mu\lambda}$ transform as the components of a connexion and a tensor resp. [2]). Therefore a geometry of the system (1.2) is the geometry of a space X_n with the connexion $\Gamma^\varkappa_{\mu\lambda}$ and the tensor $\Gamma_{\mu\lambda}$. All this is much more complicated for the analogical systems of higher orders. E. BOMPIANI [3]) studied the system of the third order.

A. MAXIA [4]) gave another geometrical interpretation of the system (1.2). If the z^α; $\alpha = 0, 1, \ldots, n$, are affine coordinates in an affine space E_{n+1}, the $n + 1$ solutions $z^\alpha = z^\alpha(\xi^\varkappa)$ of the system (1.2) represent a hypersurface A_n in E_{n+1} for which the system (1.2) together with $\partial_\lambda z = z_\lambda$ may be considered as a fundamental system. For this A_n, $\Gamma_{\mu\lambda}$ is the fundamental tensor and the $\Gamma^\varkappa_{\mu\lambda}$'s are the coefficients of the induced connexion. A. MAXIA proved that the same hypersurface A_n considered as

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[1]) The numbers in [ ] refer to the bibliography at the end of this paper.

[2]) Cf. [1].

[3]) [2].

[4]) [5].

an envelope of its tangent-E_n's is represented by the conjugated system of the system (1. 2) introduced by L. BIANCHI [5]).

In the present paper still another geometrical interpretation of the system (1. 2) is given. This system is very closely connected with the system (3. 2). Every solution of (3. 2) may be considered as a geodesic hypersurface of a space X_{n+1} with the projective connexion (3. 15). In the sections **1—6** this geometrical interpretation is developed.

The most general transformation which leaves the form of the system (1. 2) unchanged is given by (2. 8). § **7** is closely connected with the investigations of A. MAXIA and deals more in detail with the functions $\Gamma^{\varkappa}_{\mu\lambda}$ and $\Gamma_{\mu\lambda}$.

In the § 8 the conjugated system of L. BIANCHI of the system (1. 2) is constructed by means of geometrical considerations.

2. Let be given a completely integrable system of linear partial differential equations of the second order

$$\partial^2_{\mu\lambda} z = \Gamma^{\varkappa}_{\mu\lambda}\partial_{\varkappa} z + \Gamma_{\mu\lambda} z\,; \quad \varkappa,\lambda,\mu,\nu,\varrho = 1,\ldots,n, \quad . \quad . \quad (2.1)$$

in the unknown variable z and the independent variables $\xi^{\varkappa}$. We suppose the coefficients $\Gamma^{\varkappa}_{\mu\lambda} = \Gamma^{\varkappa}_{\lambda\mu}$, $\Gamma_{\mu\lambda} = \Gamma_{\lambda\mu}$ to be analytic functions of the $\xi^{\varkappa}$ only. According to the assumption that the system (2. 1) is completely integrable, these coefficients are related by the following equations

$$\left.\begin{aligned} &\text{a)}\quad \partial_{[\nu}\Gamma^{\varkappa}_{\mu]\lambda} + \Gamma^{\varkappa}_{\varrho[\nu}\Gamma^{\varrho}_{\mu]\lambda} + A^{\varkappa}_{[\nu}\Gamma_{\mu]\lambda} = 0,\\ &\text{b)}\quad \partial_{[\nu}\Gamma_{\mu]\lambda} + \Gamma_{\varrho[\nu}\Gamma^{\varrho}_{\mu]\lambda} = 0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2.2)$$

The system (2. 1) is invariant, i.e. it transforms into

$$\partial^2_{\mu'\lambda'} z' = \Gamma^{\varkappa'}_{\mu'\lambda'}\partial_{\varkappa'} z' + \Gamma_{\mu'\lambda'} z'\,; \quad \varkappa',\lambda',\mu',\nu',\varrho' = 1',\ldots,n' \quad . \quad (2.3)$$

if and only if the $\xi^{\varkappa'}$ depend on the $\xi^{\varkappa}$ only

$$\xi^{\varkappa'} = \xi^{\varkappa'}(\xi^{\varkappa}), \quad \det.\left(\frac{\partial\,\xi^{\varkappa'}}{\partial\,\xi^{\varkappa}}\right) \neq 0 \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

and if z transforms as follows

$$z' = \varrho(\xi^{\varkappa})\,z + \sigma(\xi^{\varkappa})\,; \quad \varrho(\xi^{\varkappa}) \neq 0, \quad . \quad . \quad . \quad . \quad . \quad (2.5)$$

where $\sigma(\xi^{\varkappa})$ is a solution of (2. 3). The relations between the coefficients in (2. 1) and (2. 3) are given by

$$\Gamma^{\varkappa'}_{\mu'\lambda'} = \Gamma^{\varkappa}_{\mu\lambda}\,A^{\mu\,\lambda}_{\mu'\lambda'\varkappa} + A^{\varkappa'}_{\varkappa}\,\partial_{\mu'}A^{\varkappa}_{\lambda'} + 2\,A^{\varkappa'}_{(\mu'}\partial_{\lambda')}\log\varrho \quad . \quad . \quad (2.6)$$

and

$$\left.\begin{aligned} \Gamma_{\mu'\lambda'} = \Gamma_{\mu\lambda}\,A^{\mu\,\lambda}_{\mu'\lambda'} - \Gamma^{\varkappa}_{\mu\lambda}\,A^{\mu\,\lambda\,\varkappa'}_{\mu'\lambda'\varkappa}\,\partial_{\varkappa'}\log\varrho - (\partial_{\mu'}A^{\varkappa}_{\lambda'})\,A^{\varkappa'}_{\varkappa}\,\partial_{\varkappa'}\log\varrho -\\ - (\partial_{\mu'}\log\varrho)\,(\partial_{\lambda'}\log\varrho) + \partial^2_{\mu'\lambda'}\log\varrho\,. \end{aligned}\right\} \quad (2.7)$$

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[5]) E. BOMPIANI [2] gave a similar geometrical interpretation for the analogical systems of the third order. The systems of arbitrary orders of this type may be treated in the same way.

As in (2. 5) the addition of a solution $\sigma(\xi^\varkappa)$ of the system (2. 3) may be considered as a trivial transformation, we restrict ourselves in the following to the transformations (2. 4) and (2. 5) with $\sigma = 0$.

$$\left.\begin{aligned} &\text{a)}\quad \xi^{\varkappa'} = \xi^{\varkappa'}(\xi^\varkappa),\ \det.\left(\frac{\partial \xi^{\varkappa'}}{\partial \xi^\varkappa}\right) \neq 0, \\ &\text{b)}\quad z' = \varrho(\xi^\varkappa)\, z,\ \ \varrho(\xi^\varkappa) \neq 0. \end{aligned}\right\} \quad \ldots \quad (2.8)$$

3. In order to find a geometrical interpretation of the system (2. 1) let us consider a space X_{n+1} with the coordinates $z, \xi^\varkappa$. If in this space we introduce a new coordinate

$$\xi^0 \overset{\text{def}}{=} -\log z \quad \ldots \quad (3.1)$$

instead of z, the form of (2. 1) is not invariant and (2. 1) is transformed into

$$\partial^2_{\mu\lambda}\xi^0 - (\partial_\mu \xi^0)(\partial_\lambda \xi^0) = \Gamma^\varkappa_{\mu\lambda}\,\partial_\varkappa \xi^0 - \Gamma_{\mu\lambda} \quad \ldots \quad (3.2)$$

It is evident that by means of (3. 1) to every solution of (2. 1) with the exception of the trivial solution $z = 0$ there corresponds one and only one solution of (3. 2) and conversely. Hence also (3. 2) is completely integrable if (2. 2) holds.

$$\left.\begin{aligned} &\text{a)}\quad \xi^{0'} = \xi^0 - \log \varrho(\xi^\varkappa),\ \xi^{\varkappa'} = \xi^{\varkappa'}(\xi^\varkappa);\ \det.\left(\frac{\partial \xi^{\varkappa'}}{\partial \xi^\varkappa}\right) \neq 0, \\ &\text{b)}\quad \xi^0 = \xi^{0'} + \log \varrho(\xi^\varkappa(\xi^{\varkappa'})),\ \xi^\varkappa = \xi^\varkappa(\xi^{\varkappa'});\ \varrho(\xi^\varkappa) \neq 0, \\ &\qquad\qquad \varkappa = 1, \ldots, n;\ \ \varkappa' = 1', \ldots, n', \end{aligned}\right\} \quad . \quad (3.3)$$

corresponding to (2. 8) and leaving (3. 2) invariant are in the space X_{n+1} with the coordinates ξ^α; $\alpha, \beta, \gamma, \delta = 0, 1, \ldots, n$, the allowable coordinate transformations. Hence the X_{n+1} of $\xi^0, \xi^\varkappa$ is not an ordinary X_{n+1} but an X_{n+1} with a restricted group of coordinate transformations.

For the transformations (3. 3) $\Gamma^\varkappa_{\mu\lambda}$ and $\Gamma_{\mu\lambda}$ transform according to (2. 6, 7).

The mixed components of the unity affinor are

$$\left.\begin{aligned} &A^{0'}_0 = \frac{\partial \xi^{0'}}{\partial \xi^0} = 1,\ A^{0'}_\lambda = \frac{\partial \xi^{0'}}{\partial \xi^\lambda} = -\partial_\lambda \log \varrho,\ A^{\varkappa'}_0 = \frac{\partial \xi^{\varkappa'}}{\partial \xi^0} = 0,\ A^{\varkappa'}_\varkappa = \frac{\partial \xi^{\varkappa'}}{\partial \xi^\varkappa}, \\ &A^0_{0'} = \frac{\partial \xi^0}{\partial \xi^{0'}} = 1,\ A^0_{\lambda'} = \frac{\partial \xi^0}{\partial \xi^{\lambda'}} = \partial_{\lambda'} \log \varrho,\ A^\varkappa_{0'} = \frac{\partial \xi^\varkappa}{\partial \xi^{0'}} = 0,\ A^\varkappa_{\varkappa'} = \frac{\partial \xi^\varkappa}{\partial \xi^{\varkappa'}}. \end{aligned}\right\} \quad (3.4)$$

According to the fact that the group of coordinate transformations in X_{n+1} is a restricted one there exists in X_{n+1} 1° an invariant congruence of curves

$$\xi^\varkappa = \text{const.} \quad \ldots \quad (3.5)$$

(individual curves of this congruence need not be invariant) and 2° a class of individually invariant contravariant vectors characterized by the vanishing of all components except the 0-component.

Now let ${}^*\Gamma^{\alpha}_{\gamma\beta}$ be an arbitrary symmetric linear connexion given in X_{n+1}. As we may easily prove, for the transformations (3.3) the coefficients of this connexion transform as follows

$$\left.\begin{aligned} {}^*\Gamma^{\varkappa'}_{\mu'\lambda'} = {}^*\Gamma^{\varkappa}_{\mu\lambda}\, A^{\mu\,\lambda\,\varkappa'}_{\mu'\,\lambda'\,\varkappa} + 2\,{}^*\Gamma^{\varkappa}_{\mu 0}\, A^{\varkappa'\mu}_{\varkappa(\mu'}\,\partial_{\lambda')}\log\varrho + \\ + {}^*\Gamma^{\varkappa}_{00}\, A^{\varkappa'}_{\varkappa}\,(\partial_{\mu'}\log\varrho)\,(\partial_{\lambda'}\log\varrho) + A^{\varkappa'}_{\varkappa}\,\partial_{\mu'}\,A^{\varkappa}_{\lambda'}, \end{aligned}\right\} \tag{3.6}$$

$$\left.\begin{aligned} {}^*\Gamma^{0'}_{\mu'\lambda'} = {}^*\Gamma^{0}_{\mu\lambda}\, A^{\mu\,\lambda}_{\mu'\lambda'} + 2\,{}^*\Gamma^{0}_{\mu 0}\, A^{\mu}_{(\mu'}\,\partial_{\lambda')}\log\varrho + {}^*\Gamma^{0}_{00}\,(\partial_{\mu'}\log\varrho)\,(\partial_{\lambda'}\log\varrho) - \\ - [{}^*\Gamma^{\varkappa}_{\mu\lambda}\, A^{\mu\,\lambda\,\varkappa'}_{\mu'\lambda'\varkappa} + 2\,{}^*\Gamma^{\varkappa}_{\mu 0}\, A^{\varkappa'\mu}_{\varkappa(\mu'}\,\partial_{\lambda')}\log\varrho + {}^*\Gamma^{\varkappa}_{00}\, A^{\varkappa'}_{\varkappa}\,(\partial_{\mu'}\log\varrho)\,(\partial_{\lambda'}\log\varrho) + \\ + A^{\varkappa'}_{\varkappa}\,\partial_{\mu'}\,A^{\varkappa}_{\lambda'}]\cdot\partial_{\varkappa'}\log\varrho + \partial^2_{\mu'\lambda'}\log\varrho, \end{aligned}\right\} \tag{3.7}$$

$$ {}^*\Gamma^{\alpha'}_{\gamma' 0'} = {}^*\Gamma^{\alpha}_{\gamma 0}\, A^{\gamma\,\alpha'}_{\gamma'\alpha}\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{3.8}$$

$$\varkappa, \lambda, \mu = 1, \ldots, n \quad ; \quad \varkappa', \lambda', \mu' = 1', \ldots, n',$$
$$\alpha, \beta, \gamma = 0, 1, \ldots, n \quad ; \quad \alpha', \beta', \gamma' = 0', 1', \ldots, n'.$$

In our A_{n+1} (i.e. in the X_{n+1} with the connexion ${}^*\Gamma^{\alpha}_{\gamma\beta}$) there exists a well defined set of X_n's whose equations are the solutions

$$\xi^0 = f(\xi^{\varkappa}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{3.9}$$

of the system (3.2). The question arises whether there exist any connexions ${}^*\Gamma^{\alpha}_{\gamma\beta}$ in A_{n+1} for which all these X_n's are geodesic X_n's.

It is easily proved that (3.9) represents a geodesic X_n in X_{n+1} if and only if the function $f(\xi^{\varkappa})$ is a solution of the system

$$\left.\begin{aligned} \partial^2_{\mu\lambda} f + ({}^*\Gamma^{0}_{00}\, A^{\varkappa}_{(\mu} - 2\,{}^*\Gamma^{\varkappa}_{(\mu|0|})\,(\partial_{\lambda)} f)\,(\partial_{\varkappa} f) - \\ - {}^*\Gamma^{\varkappa}_{00}\,(\partial_{\mu} f)\,(\partial_{\lambda} f)\,(\partial_{\varkappa} f) - ({}^*\Gamma^{\varkappa}_{\mu\lambda} - 2\,{}^*\Gamma^{0}_{(\mu|0|}\, A^{\varkappa}_{\lambda)})\,\partial_{\varkappa} f + {}^*\Gamma^{0}_{\mu\lambda} = 0. \end{aligned}\right\} \tag{3.10}$$

Accordingly every solution of (3.2) has to be a solution of (3.10). But because of the complete integrability of (3.2) this is possible if and only if (3.2) and (3.10) are equivalent. Hence

$$\left.\begin{aligned} &\text{a)}\ {}^*\Gamma^{\varkappa}_{00} = 0\,, && \text{b)}\ {}^*\Gamma^{0}_{\mu\lambda} = \Gamma_{\mu\lambda}\,, \\ &\text{c)}\ {}^*\Gamma^{\varkappa}_{\mu\lambda} = \Gamma^{\varkappa}_{\mu\lambda} + 2\,{}^*\Gamma^{0}_{(\mu|0|}\, A^{\varkappa}_{\lambda)}\,, && \text{d)}\ {}^*\Gamma^{\varkappa}_{\mu 0} = \tfrac{1}{2}\,(1 + {}^*\Gamma^{0}_{00})\, A^{\varkappa}_{\mu}. \end{aligned}\right\} \tag{3.11}$$

From (3.11) we see that the connexion ${}^*\Gamma^{\alpha}_{\gamma\beta}$ is not determined uniquely. The ${}^*\Gamma^{0}_{\gamma 0}$'s may be still chosen arbitrarily but they must transform according to (3.8) and therefore

$${}^*\Gamma^{0'}_{0'0'} = {}^*\Gamma^{0}_{00}\,, \quad {}^*\Gamma^{0'}_{\mu' 0'} = {}^*\Gamma^{0}_{\mu 0}\, A^{\mu}_{\mu'} + \tfrac{1}{2}\,(-1 + {}^*\Gamma^{0}_{00})\,\partial_{\mu'}\log\varrho\,. \tag{3.12}$$

If we write now

$$p_0 \overset{\text{def}}{=} \tfrac{1}{2}\,(-1 + {}^*\Gamma^{0}_{00}), \quad p_{\lambda} \overset{\text{def}}{=} {}^*\Gamma^{0}_{\lambda 0}\,, \quad . \quad . \quad . \quad . \quad . \quad . \tag{3.13}$$

we see that p_{β}, $\beta = 0, 1, \ldots, n$, is a covariant vector. Conversely, if p_{β} is

an arbitrary covariant vector, all connexions satisfying (3.11) may be written in the form

$$\left.\begin{aligned} &{}^*\Gamma^{\varkappa}_{00}=0,\ {}^*\Gamma^{\varkappa}_{\mu 0}=A^{\varkappa}_{\mu}+p_0 A^{\varkappa}_{\mu},\ {}^*\Gamma^{\varkappa}_{\mu\lambda}=\Gamma^{\varkappa}_{\mu\lambda}+2p_{(\mu}A^{\varkappa}_{\lambda)},\\ &{}^*\Gamma^{0}_{00}=1+2p_0,\ {}^*\Gamma^{0}_{\mu 0}=p_{\mu},\ {}^*\Gamma^{0}_{\mu\lambda}=\Gamma_{\mu\lambda}, \end{aligned}\right\} \quad (3.14)$$

or, taking all equations together

$${}^*\Gamma^{\alpha}_{\gamma\beta}={}^0\Gamma^{\alpha}_{\gamma\beta}+2p_{(\gamma}A^{\alpha}_{\beta)}\ :\quad \alpha,\beta,\gamma=0,1,\ldots,n, \quad . \quad . \quad (3.15)$$

where the connexion ${}^0\Gamma^{\alpha}_{\gamma\beta}$ is defined by

$$\text{a)}\ {}^0\Gamma^{\varkappa}_{\mu\lambda}\overset{\text{def}}{=}\Gamma^{\varkappa}_{\mu\lambda}, \qquad \text{b)}\ {}^0\Gamma^{0}_{\mu\lambda}\overset{\text{def}}{=}\Gamma_{\mu\lambda}, \qquad \text{c)}\ {}^0\Gamma^{\alpha}_{\gamma 0}\overset{\text{def}}{=}A^{\alpha}_{\gamma}. \quad (3.16)$$

(3.15) is a so called projective transformation of the connexion, i.e. a transformation leaving the set of all geodesics invariant.

Hence we have proved:

Theorem I. *A necessary and sufficient condition that every solution* $\xi^0=f(\xi^{\varkappa})$ *of* (3.2) *represents a geodesic* X_n *in the* A_{n+1} *with the connexion* ${}^*\Gamma^{\alpha}_{\gamma\beta}$ *is that* (3.11) *be satisfied. The connexion* ${}^*\Gamma^{\alpha}_{\gamma\beta}$ *is determined to within the projective transformation* (3.15).

All connexions (3.15) define a so called projective connexion. The projective curvature affinor $P_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}$ of this projective connexion is [6])

$$P_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}\overset{\text{def}}{=}{}^*R_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}-2P_{[\delta\gamma]}A^{\alpha}_{\beta}+2A^{\alpha}_{[\delta}P_{\gamma]\beta} \quad . \quad . \quad . \quad (3.17)$$

where

$$\left.\begin{aligned} &P_{\gamma\beta}\overset{\text{def}}{=}\frac{1}{1-(n+1)^2}\{(n+2)\,{}^*R_{\gamma\beta}+{}^*V_{\gamma\beta}\},\\ &{}^*R_{\gamma\beta}\overset{\text{def}}{=}{}^*R_{\alpha\dot\gamma\dot\beta}{}^{\alpha},\quad {}^*V_{\gamma\beta}\overset{\text{def}}{=}{}^*R_{\gamma\dot\beta\dot\alpha}{}^{\alpha} \end{aligned}\right\} \quad . \quad . \quad (3.18)$$

and where ${}^*R_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}$ is the curvature affinor of the connexion ${}^*\Gamma^{\alpha}_{\gamma\beta}$. It is well known that $P_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}$ is independent of the choice of p_β. For the connexion ${}^0\Gamma^{\alpha}_{\gamma\beta}$ we may find easily the corresponding curvature affinor ${}^0R_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}$. All its components except

$$\left.\begin{aligned} &\text{a)}\ \tfrac{1}{2}\,{}^0R_{\dot\nu\dot\mu\dot\lambda}{}^{\varkappa}=\partial_{[\nu}\Gamma^{\varkappa}_{\mu]\lambda}+\Gamma^{\varkappa}_{\varrho[\nu}\Gamma^{\varrho}_{\mu]\lambda}+A^{\varkappa}_{[\nu}\Gamma_{\mu]\lambda},\\ &\text{b)}\ \tfrac{1}{2}\,{}^0R_{\dot\nu\dot\mu\dot\lambda}{}^{0}=\partial_{[\nu}\Gamma_{\mu]\lambda}+\Gamma_{\varrho[\nu}\Gamma^{\varrho}_{\mu]\lambda} \end{aligned}\right\} \quad . \quad . \quad (3.19)$$

vanish identically. But the components (3.19) vanish in consequence of (2.2), hence

$${}^0R_{\dot\delta\dot\gamma\dot\beta}{}^{\alpha}=0. \ [7]) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.20)$$

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[6]) Cf. [7], II, p. 179.

[7]) The conditions of integrability (2.2) of the system (2.1) can be therefore replaced by the equation (3.20).

From this result and (3.17) we obtain:

Theorem II. *The projective curvature affinor of the projective connexion* (3.15) *vanishes*

$$P_{\delta\gamma\beta}{}^{\alpha} = 0. \qquad (3.21)$$

We remark that according to (3.20) there exists in the X_{n+1} a coordinate system (α') such that the ${}^{0}\Gamma^{\alpha'}_{\gamma'\beta'}$ all vanish. But according to (3.16c) the transformation $(\alpha) \to (\alpha')$ does not satisfy the conditions for allowable transformations (3.3) and therefore it does not leave invariant the form of the equations (3.2).

4. The connexions (3.15) are determined by the only geometric condition that all solutions $\xi^0 = f(\xi^x)$ of the system (3.2) represent geodesic X_n's of the connexion ${}^{*}\Gamma^{\alpha}_{\gamma\beta}$.

Some of the connexions (3.15)

$${}^{*}\Gamma^{\alpha}_{\gamma\beta} = {}^{0}\Gamma^{\alpha}_{\gamma\beta} + 2p_{(\gamma}A^{\alpha}_{\beta)} \qquad (4.1)$$

may be distinguished by introducing an additional geometric condition. We may ask for connexions of the form (4.1) for which the invariant congruence (3.5)

$$\xi^x = \text{const.} \qquad (4.2)$$

consists of pseudoparallel geodesics. A necessary and sufficient condition is that there exists a vector field v^{α} with $v^x = 0$ such that $\delta v^{\alpha} = \varepsilon v^{\alpha}$ or, according to (4.1) and (3.16)

$$\left.\begin{aligned} &{}^{*}\Gamma^{x}_{00}\, v^0 d\xi^0 + {}^{*}\Gamma^{x}_{\mu 0}\, v\ d\xi^{\mu} = (1+p_0)\, v^0 d\xi^x = 0, \\ &dv^0 + {}^{*}\Gamma^{0}_{00} v^0 d\xi^0 + {}^{*}\Gamma^{0}_{\mu 0} v^0 d\xi^{\mu} = dv^0 + (1+2p_0) v^0 d\xi^0 + v^0 p_{\mu} d\xi^{\mu} = \varepsilon v^0. \end{aligned}\right\} \qquad (4.3)$$

Hence the curves (4.2) are always geodesics but the only values of p_{β} for which they are pseudoparallel are those for which

$$p_0 = -1. \qquad (4.4)$$

This condition is invariant for the transformations (3.3).

Collecting results we have:

Theorem III. *The curves of the invariant congruence* (4.2) *are always geodesics of the connexion* ${}^{*}\Gamma^{\alpha}_{\gamma\beta}$*; they are pseudoparallel if and only if the invariant condition* $p_0 = -1$ *is satisfied.*

We remark that the connexion ${}^{0}\Gamma^{\alpha}_{\gamma\beta}$ does not possess this property as ${}^{*}\Gamma^{\alpha}_{\gamma\beta} = {}^{0}\Gamma^{\alpha}_{\gamma\beta}$ only for $p_{\beta} = 0$ which equation is invariant for the transformations (3.3).

In order to distinguish certain connexions from (4.1), also other additional geometric conditions can be introduced. If $\xi^0 = f(\xi^x)$ is a solution of (3.2), also $\xi^0 = f(\xi^x) + c$ ($c = \text{const.}$) is a solution. We remark that $\xi^0 = \text{const.}$ is not a solution, except in the case that $\Gamma_{\mu\lambda} = 0$.

All the X_n's

$$\xi^0 = f(\xi^\varkappa) + c \quad ; \quad c = \text{const.}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.5)$$

are always geodesic X_n's of the connexions (4.1). The solution $\xi^0 = f(\xi^\varkappa)$ being given we may ask for the connexions of the form (4.1) for which these X_n's are pseudoparallel.

The field

$$t_0 = -1, \quad t_\lambda = \partial_\lambda f \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.6)$$

is tangent to the X_n's (4.5). The X_n's being geodesic we have only to consider an arbitrary displacement not lying in the n-direction of t_β, e.g. a displacement with $d\xi^0 \neq 0$, $d\xi^\varkappa = 0$. Then for this displacement we have to write out the equation

$$d\xi^\lambda \, {}^*\nabla_\lambda t_\beta = \varepsilon t_\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.7)$$

where ε is an arbitrary scalar (${}^*\nabla_\lambda$ symbolizis the covariant derivation with respect to ${}^*\Gamma^\alpha_{\gamma\beta}$). According to (4.1) and (3.16) this gives immediately

$$p_\lambda + p_0 \partial_\lambda f = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.8)$$

The general solution of this equation is

$$p_0 = -\sigma, \quad p_\lambda = \sigma \partial_\lambda f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.9)$$

where σ is an arbitrary scalar. Hence we have proved:

Theorem IV. *The arbitrarily but definitely chosen solutions of* (3.2)

$$\xi^0 = f(\xi^\varkappa) + c \quad ; \quad c = \text{const.}, \quad . \quad . \quad . \quad . \quad . \quad (4.10)$$

represent always geodesic X_n's of the connexion (4.1). *These X_n's are pseudoparallel if and only if*

$$p_0 = -\sigma, \quad p_\lambda = \sigma \partial_\lambda f \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.11)$$

where σ is an arbitrary scalar.

From the last two theorems follows immediately:

Theorem V. *If*

$$\xi^0 = f(\xi^\varkappa) + c \quad ; \quad c = \text{const.}, \quad . \quad . \quad . \quad . \quad . \quad (4.12)$$

are ∞^1 arbitrarily but definitely chosen solutions of (3.2), *the ∞^1 X_n's represented by* (4.12) *are always geodesic for connexions of the form* (4.1). *There exists one and only one among these connexions such that these X_n's are pseudoparallel and that simultaneously the curves of the invariant congruence of geodesics* (4.2) *are pseudoparallel. This connexion is given by* (4.1) *and*

$$p_0 = -1, \quad p_\lambda = \partial_\lambda f. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.13)$$

5. The equation (3.20) shows that the curvature affinor ${}^0R^{\cdot\cdot\cdot\alpha}_{\delta\gamma\beta}$ of the connexion ${}^0\Gamma^\alpha_{\gamma\beta}$ vanishes; the space A_{n+1} with this connexion is therefore an E_{n+1}. In order to find all the connexions ${}^*\Gamma^\alpha_{\gamma\beta}$ the curvature affinor of

which vanishes we consider the transformation formula of the curvature affinor for the transformation (3.15). According to ${}^0R_{\partial\gamma\beta}^{\ \ \ \alpha} = 0$ we obtain

$$ {}^*R_{\partial\gamma\beta}^{\ \ \ \alpha} = -2p_{[\partial\gamma]}A_{\beta}^{\alpha} + 2A_{[\partial\beta}^{\alpha}p_{\gamma]\beta} \quad . \quad . \quad . \quad . \quad . \quad (5.1) $$

where

$$ p_{\gamma\beta} \overset{\text{def}}{=} -{}^0\nabla_{\gamma}p_{\beta} + p_{\gamma}p_{\beta}\,, \quad [8]) \quad . \quad . \quad . \quad . \quad . \quad (5.2) $$

${}^0\nabla_{\beta}$ being the symbol of the covariant derivation with respect to ${}^0\Gamma_{\gamma\beta}^{\alpha}$. A necessary and sufficient condition that ${}^*R_{\partial\gamma\beta}^{\ \ \ \alpha} = 0$ is that p_{β} is a solution of the system of differential equations

$$ {}^0\nabla_{\gamma}p_{\beta} - p_{\gamma}p_{\beta} = 0. \quad [9]) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.3) $$

According to (3.16) this system may be written as follows

$$ \left.\begin{array}{ll} \text{a)}\ \partial_0 p_0 - p_0 - p_0 p_0 = 0\,, & \text{b)}\ \partial_\lambda p_0 - p_\lambda - p_\lambda p_0 = 0\,, \\ \text{c)}\ \partial_0 p_\lambda - p_\lambda - p_\lambda p_0 = 0\,, & \text{d)}\ \partial_\mu p_\lambda - p_\mu p_\lambda = \Gamma_{\mu\lambda}^{\varkappa} p_\varkappa + \Gamma_{\mu\lambda} p_0. \end{array}\right\} \quad (5.4) $$

First we consider the solutions p_{β} for which $p_0 = \text{const}$. We may easily verify that all such solutions are given by

$$ \left.\begin{array}{l} \text{a)}\ \ p_0 = 0\,, \quad p_\lambda = 0\,, \\ \text{b)}\ \ p_0 = -1\,, \quad p_\lambda = p_\lambda(\xi^\varkappa)\,, \quad \partial_\mu p_\lambda - p_\mu p_\lambda = \Gamma_{\mu\lambda}^{\varkappa} p_\varkappa - \Gamma_{\mu\lambda}\,. \end{array}\right\} \quad (5.5) $$

Hence we have proved:

Theorem VI. *The connexions* ${}^*\Gamma_{\gamma\beta}^{\alpha}$ *from* (4.1), *the curvature affinor* ${}^*R_{\partial\gamma\beta}^{\ \ \ \alpha}$ *of which vanishes and for which* $p_0 = \text{const.}$, *are either* ${}^0\Gamma_{\gamma\beta}^{\alpha}$ *or the connexions for which the congruence*

$$ \xi^{\varkappa} = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.6) $$

consists of pseudoparallel geodesics.

Now let us suppose $p_0 \neq 0$. According to (5.5) this is equivalent to ${}^*\Gamma_{\gamma\beta}^{\alpha} \neq {}^0\Gamma_{\gamma\beta}^{\alpha}$. In this case we may replace the system (5.4) by

$$ \left.\begin{array}{l} \text{a)}\ \partial_0 p_0 - p_0 - p_0 p_0 = 0\,; \quad p_0 \neq 0\,, \\ \text{b)}\ \partial_\lambda p_0 - p_\lambda - p_\lambda p_0 = 0\,, \\ \text{c)}\ \partial_0 p_\lambda - p_\lambda - p_\lambda p_0 = 0\,, \\ \text{d)}\ \partial_\mu P_\lambda - P_\mu P_\lambda = \Gamma_{\mu\lambda}^{\varkappa} P_\varkappa - \Gamma_{\mu\lambda} \quad ; \quad P_\lambda \overset{\text{def}}{=} -\dfrac{p_\lambda}{p_0}\,. \end{array}\right\} \quad . \quad . \quad (5.7) $$

Hence:

Theorem VII. *A necessary and sufficient condition that the curvature affinor* ${}^*R_{\partial\gamma\beta}^{\ \ \ \alpha}$ *of a connexion* ${}^*\Gamma_{\gamma\beta}^{\alpha}$ $(\neq {}^0\Gamma_{\gamma\beta}^{\alpha})$ *from* (4.1) *vanishes is that* p_{β} *is a solution of* (5.7).

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[8]) Cf. [7], II, p. 177.
[9]) Cf. [7], II, p. 182.

6. The solutions of the system (3. 2) represent geodesic X_n's in the A_{n+1} with the connexion (4. 1). Conversely, according to the complete integrability of (3. 2) every geodesic X_n of this connexion with an equation of the form (3. 9), i.e. every geodesic X_n that has nowhere a direction in common with the invariant congruence (3. 5)

$$\xi^x = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.1)$$

represents a solution of (3. 2). Every other geodesic X_n consists of ∞^{n-1} curves of this congruence and its equation has the form

$$f(\xi^x) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.2)$$

with the condition

$$d\xi^r \, {}^*\nabla_\lambda \, \partial_\beta f = \sigma \, \partial_\beta f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.3)$$

for every direction $d\xi^\alpha$ for which

$$d\xi^x \, \partial_x f = 0, \quad d\xi_0 \text{ arbitrary}, \quad . \quad . \quad . \quad . \quad . \quad (6.4)$$

σ being an arbitrary scalar. From (6. 3) it follows that

$$d\xi^\mu \, (\partial^2_{\mu\lambda} f - {}^*\Gamma^x_{\mu\lambda} \, \partial_x f) = \sigma' \, \partial_\lambda f \quad . \quad . \quad . \quad . \quad . \quad (6.5)$$

where σ' is an arbitrary scalar.

Because of (6. 4) the equation (6. 5) is equivalent to

$$\partial^2_{\mu\lambda} f = ({}^*\Gamma^x_{\mu\lambda} + 2 \, {}^*a_{(\mu} A^x_{\lambda)}) \, \partial_x f \quad . \quad . \quad . \quad . \quad . \quad (6.6)$$

or in another form

$$\partial^2_{\mu\lambda} f = (\Gamma^x_{\mu\lambda} + 2 \, a_{(\mu} A^x_{\lambda)}) \, \partial_x f \quad . \quad . \quad . \quad . \quad . \quad (6.7)$$

where a_μ is an arbitrary vector and

$$a_\lambda = {}^*a_\lambda + p_\lambda . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.8)$$

The conditions of integrability of the system (6. 7) are satisfied if and only if a_λ is a gradient of some scalar a

$$a_\lambda = \partial_\lambda a . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.9)$$

That proves:

Theorem VIII. *If $f = f(\xi^x)$ is a solution of the equations*

$$\partial^2_{\mu\lambda} f = (\Gamma^x_{\mu\lambda} + 2 \, a_{(\mu} A^x_{\lambda)}) \, \partial_x f \quad . \quad . \quad . \quad . \quad . \quad (6.10)$$

where $a_\mu = \partial_\mu a$ and a an arbitrary scalar, then $f = 0$ is the equation of a geodesic X_n for every connexion (3. 15).

7. In this section we consider the functions $\Gamma^x_{\mu\lambda}$ and $\Gamma_{\mu\lambda}$ in detail. Instead of the group of transformations (2. 8) we consider firstly only the subgroup

$$\text{a)} \quad \xi^{0'} = \xi^0, \qquad \text{b)} \quad \xi^{x'} = \xi^{x'}(\xi^r); \quad \det. \left(\frac{\partial \xi^{x'}}{\partial \xi^x} \right) \neq 0 . \quad . \quad (7.1)$$

Then the formulae (2.6) and (2.7) simplify and the resulting equations are

$$\Gamma^{\varkappa'}_{\mu'\lambda'} = \Gamma^{\varkappa}_{\mu\lambda} A^{\mu\lambda\varkappa'}_{\mu'\lambda'\varkappa} + A^{\varkappa'}_{\varkappa} \partial_{\mu'} A^{\varkappa}_{\lambda'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.2)$$

$$\Gamma_{\mu'\lambda'} = \Gamma_{\mu\lambda} A^{\mu\lambda}_{\mu'\lambda'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.3)$$

from which it follows that $\Gamma^{\varkappa}_{\mu\lambda}$ resp. $\Gamma_{\mu\lambda}$ are the components of a connexion and a tensor resp. in the X_n of the $\xi^{\varkappa}$. If $R^{\cdot\cdot\cdot\varkappa}_{\nu\mu\lambda}$ is the curvature affinor of the connexion $\Gamma^{\varkappa}_{\mu\lambda}$ and ∇_{λ} the symbol of the covariant derivation with respect to $\Gamma^{\varkappa}_{\mu\lambda}$, the conditions of integrability (2.2) are equivalent to

$$\text{a)} \quad R^{\cdot\cdot\cdot\varkappa}_{\nu\mu\lambda} = 2\,\Gamma_{\lambda[\nu} A^{\varkappa}_{\mu]}, \qquad \text{b)} \quad \nabla_{[\nu}\Gamma_{\mu]\lambda} = 0. \ ^{10}) \quad . \quad . \quad (7.4)$$

If we define $R_{\mu\lambda} \overset{\text{def}}{=} R^{\cdot\cdot\cdot\varkappa}_{\varkappa\mu\lambda}$, $V_{\mu\lambda} \overset{\text{def}}{=} R^{\cdot\cdot\cdot\varkappa}_{\mu\lambda\varkappa}$ we find from (7.4)

$$R_{\mu\lambda} = (1-n)\,\Gamma_{\mu\lambda}, \quad V_{\mu\lambda} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.5)$$

If now we apply to the unknown variable the transformation (cf. (3.3))

$$\overset{\varrho}{\xi}{}^{0} = \xi^{0} - \log \varrho(\xi^{\varkappa}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.6)$$

and denote the transforms of $\Gamma^{\varkappa}_{\mu\lambda}$ and $\Gamma_{\mu\lambda}$ by $\overset{\varrho}{\Gamma}{}^{\varkappa}_{\mu\lambda}$ and $\overset{\varrho}{\Gamma}_{\mu\lambda}$ resp., according to (2.6,7) and (7.2,3) we obtain

$$\overset{\varrho}{\Gamma}{}^{\varkappa}_{\mu\lambda} = \Gamma^{\varkappa}_{\mu\lambda} + 2\,A^{\varkappa}_{(\mu}\partial_{\lambda)} \log \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.7)$$

$$\overset{\varrho}{\Gamma}_{\mu\lambda} = \Gamma_{\mu\lambda} - \Gamma^{\varkappa}_{\mu\lambda}\partial_{\varkappa} \log \varrho - (\partial_{\mu} \log \varrho)(\partial_{\lambda} \log \varrho) + \partial^{2}_{\mu\lambda} \log \varrho. \ ^{11}) \quad (7.8)$$

By the elimination of $\partial_{\lambda} \log \varrho$ from (7.7) we get the well known projective parameters of THOMAS [12])

$$\Pi^{\varkappa}_{\mu\lambda} \overset{\text{def}}{=} \Gamma^{\varkappa}_{\mu\lambda} - \frac{2}{n+1} A^{\varkappa}_{(\mu}\,\Gamma^{\nu}_{\lambda)\nu} \ ^{13}) \quad . \quad . \quad . \quad . \quad (7.9)$$

transforming for the transformations (7.1b) and (7.6) as follows

$$\Pi^{\varkappa'}_{\mu'\lambda'} = \Pi^{\varkappa}_{\mu\lambda} A^{\mu\lambda\varkappa'}_{\mu'\lambda'\varkappa} + \frac{2}{n+1} A^{\varkappa'}_{(\mu'}\partial_{\lambda')} \log \Delta + A^{\varkappa'}_{\varkappa}\partial_{\mu'} A^{\varkappa}_{\lambda'}; \ \Delta = \det.(A^{\varkappa'}_{\varkappa}), \quad (7.10)$$

that is independently of the parameter ϱ.

If we eliminate $\log \varrho$ from (7.7) and (7.8) we obtain the functions

$$\Omega_{\mu\lambda} \overset{\text{def}}{=} \Gamma_{\mu\lambda} + \frac{1}{n+1}\left\{\Gamma^{\varkappa}_{\mu\lambda}\,\Gamma^{\nu}_{\varkappa\nu} - \left(\partial_{\mu}\,\Gamma^{\nu}_{\lambda\nu} + \frac{1}{n+1}\,\Gamma^{\nu}_{\mu\nu}\Gamma^{\varrho}_{\lambda\varrho}\right)\right\} \ ^{14}) \quad (7.11)$$

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10) Cf. [5], (4.1), p. 172.

11) Cf. [5], (7.2), p. 174.

12) Cf. [7], II, p. 193.

13) $\Pi^{\varkappa}_{\mu\lambda}$ correspond to ${}^{*}\Gamma^{t}_{rs}$ in [5], (7.3), p. 174.

14) $\Omega_{\mu\lambda}$ correspond to ${}^{*}b_{rs}$ in [5], (7.3), p. 174.

which for the transformations (7.1*b*) and (7.6) transform also independently of the parameter ϱ:

$$\left.\begin{aligned}&\Omega_{\mu'\lambda'} = \Omega_{\mu\lambda}\, A^{\mu\lambda}_{\mu'\lambda'} + \frac{1}{n+1}\,\Gamma^{\varkappa'}_{\mu'\lambda'}\,\partial_{\varkappa'}\log\Delta - \\ &- \frac{1}{(n+1)^2}\{2\,\Gamma^{\varkappa}_{\nu\mu}\,A^{\mu}_{(\mu'}\,\partial_{\lambda')}\log\Delta + (\partial_{\mu'}\log\Delta)(\partial_{\lambda'}\log\Delta) + \partial^2_{\mu'\lambda'}\log\Delta\}.\ ^{15})\end{aligned}\right\} \quad (7.12)$$

The functions $\Omega_{\mu\lambda}$, however, may be derived directly from the parameters $\Pi^{\varkappa}_{\mu\lambda}$. In order to find a relation between $\Omega_{\mu\lambda}$ and $\Pi^{\varkappa}_{\mu\lambda}$ let us introduce the following functions

$$\Pi^{\varkappa}_{\nu\mu\lambda} \overset{\text{def}}{=} 2\,\partial_{[\nu}\,\Pi^{\varkappa}_{\mu]\lambda} + 2\,\Pi^{\varkappa}_{[\nu|\varrho|}\,\Pi^{\varrho}_{\mu]\lambda}\ ^{16}) \quad . \quad . \quad . \quad . \quad (7.13)$$

$$\Phi_{\mu\lambda} \overset{\text{def}}{=} \partial_{\mu}\,\Gamma^{\varkappa}_{\lambda\varkappa} + \frac{1}{n+1}\,\Gamma^{\nu}_{\mu\nu}\,\Gamma^{\varkappa}_{\lambda\varkappa} - \Gamma^{\varkappa}_{\mu\lambda}\,\Gamma^{\nu}_{\varkappa\nu}.\ ^{17}) \quad . \quad . \quad (7.14)$$

Then we obtain from (7.9), (7.13) and (7.14)

$$\Pi^{\varkappa}_{\nu\mu\lambda} = R^{\cdot\ \cdot\ \varkappa}_{\nu\ \mu\ \lambda} + \frac{2}{n+1}\{A^{\varkappa}_{[\nu}\,\Phi_{\mu]\lambda} - A^{\varkappa}_{\lambda}\,\partial_{[\nu}\,\Gamma^{\varrho}_{\mu]\varrho}\} \quad . \quad . \quad (7.15)$$

and from this equation and (7.5) it follows that

$$\Phi_{\mu\lambda} = \frac{n+1}{n-1}\,\Pi_{\mu\lambda} + (n+1)\,\Gamma_{\mu\lambda} \quad . \quad . \quad . \quad . \quad . \quad (7.16)$$

where $\Pi_{\mu\lambda} \overset{\text{def}}{=} \Pi^{\varkappa}_{\varkappa\mu\lambda}$.

Evidently the relation between the $\Omega_{\mu\lambda}$ and the $\Pi^{\varkappa}_{\mu\lambda}$ is given by

$$\Omega_{\mu\lambda} = \frac{1}{1-n}\,\Pi_{\mu\lambda}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.17)$$

From (7.7) it is evident that to every system of the form (2.1) exists a projective curvature affinor in the X_n of the $\xi^{\varkappa}$ which, however, vanishes because of (7.4*a*).

8. In this section the unknown variable ξ^0 will not be transformed and accordingly be denoted by ζ. Then, as we have seen, in the X_n of the $\xi^{\varkappa}$ (obtained from the X_{n+1} of the ξ^{α} by reduction with respect to the invariant congruence (3.5) [18])) a connexion $\Gamma^{\varkappa}_{\mu\lambda}$ and a tensor $\Gamma_{\mu\lambda}$ are fixed. We deal here only with the case that this tensor has the rank n. In that case we may write $a_{\mu\lambda}$ instead of $\Gamma_{\mu\lambda}$ and use $a_{\mu\lambda}$ as a fundamental tensor in X_n to raise and lower indices. Of course $\Gamma^{\varkappa}_{\mu\lambda}$ is in general not

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15) From (7.10) and (7.12) it follows that the statement of the last theorem in [5], p. 174, concerning the functions $\Pi^{\varkappa}_{\mu\lambda}$ and $\Omega_{\mu\lambda}$ must be formulated in a somewhat other way.

16) Cf. [4], (35.8), p. 99.

17) Cf. [4], p. 100 where A_{ij} stands for $\Phi_{\mu\lambda}$.

18) That means: all points lying in the same $\xi^{\varkappa} = \text{const.}$ are identified. Cf. [6], p. 45.

equal to the connexion $\{{}^{\varkappa}_{\mu\lambda}\}$ belonging to this fundamental tensor. According to this fact next to the covariant differential δw_λ with respect to the connexion $\Gamma^{\varkappa}_{\mu\lambda}$ another covariant differential

$$'\delta w_\lambda \overset{\text{def}}{=} a_{\lambda\varkappa}\,\delta a^{\varkappa\nu}\,w_\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.1)$$

can be defined. The connexion $'\Gamma^{\varkappa}_{\mu\lambda}$ belonging to this second covariant differential can be formed as follows. From (8.1) it follows that

$$'\delta w_\lambda = d w_\lambda - (\Gamma^{\varkappa}_{\mu\lambda} - Q_{\mu\,\cdot\,\lambda}^{\cdot\,\varkappa})\,w_\varkappa\,d\xi^\mu \quad . \quad . \quad . \quad . \quad (8.2)$$

where

$$Q_\mu^{\cdot\,\lambda\varkappa} \overset{\text{def}}{=} \nabla_\mu a^{\lambda\varkappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.3)$$

and accordingly

$$'\Gamma^{\varkappa}_{\mu\lambda} = \Gamma^{\varkappa}_{\mu\lambda} - Q_{\mu\,\cdot\,\lambda}^{\cdot\,\varkappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.4)$$

The connexion $'\Gamma^{\varkappa}_{\mu\lambda}$ may be written also in another form using the well known [19]) formula

$$\Gamma^{\varkappa}_{\mu\lambda} = \{{}^{\varkappa}_{\mu\lambda}\} + \tfrac{1}{2}\,(Q_{\mu\,\cdot\,\lambda}^{\cdot\,\varkappa} + Q_{\lambda\,\cdot\,\mu}^{\cdot\,\varkappa} - Q^{\varkappa}_{\cdot\,\mu\lambda}) \quad . \quad . \quad . \quad . \quad (8.5)$$

In consequence of (7.4*b*)

$$Q_{[\nu\,\mu]\lambda} = \dot{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.6)$$

and therefore from (8.5) it follows that

$$Q_{\mu\,\cdot\,\lambda}^{\cdot\,\varkappa} = 2\,(\Gamma^{\varkappa}_{\mu\lambda} - \{{}^{\varkappa}_{\mu\lambda}\}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.7)$$

with aid of which we obtain from (8.4)

$$'\Gamma^{\varkappa}_{\mu\lambda} = 2\,\{{}^{\varkappa}_{\mu\lambda}\} - \Gamma^{\varkappa}_{\mu\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.8)$$

Evidently (8.8) is equivalent to

$$'\Gamma^{\varkappa}_{\mu\lambda} = a^{\varkappa\varrho}\,(\partial_\mu a_{\lambda\varrho} - \Gamma^{\nu}_{\varrho\mu}\,a_{\lambda\nu}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.9)$$

If $'\nabla_\lambda$ is the symbol of the covariant derivation with respect to the connexion $'\Gamma^{\varkappa}_{\mu\lambda}$ and if $'R_{\nu\,\cdot\,\mu\lambda}^{\cdot\,\varkappa}$ is the curvature affinor of this same connexion, it is easily proved that the following equations

$$\text{a)}\quad '{R_{\nu\,\cdot\,\mu\lambda}^{\cdot\,\varkappa}} = 2\,\Gamma_{\lambda[\nu}A^{\varkappa}_{\mu]}\,, \qquad \text{b)}\quad '\nabla_{[\nu}\Gamma_{\mu]\lambda} = 0 \quad . \quad . \quad (8.10)$$

are satisfied for the connexion $'\Gamma^{\varkappa}_{\mu\lambda}$. Therefore the functions $'\Gamma^{\varkappa}_{\mu\lambda}$ and $\Gamma_{\mu\lambda}$ may be considered as the coefficients of the following system of the partial differential equations of the second order

$$\partial^2_{\mu\lambda}\zeta = '\Gamma^{\varkappa}_{\mu\lambda}\,\partial_\varkappa\zeta + \Gamma_{\mu\lambda}\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.11)$$

in the unknown function ζ and the independent variables $\xi^\varkappa$. The conditions of integrability of this system are (8.10). The system (8.11), however, is the well known conjugated system to the system (2.1) introduced by

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[19]) Cf. [7], I, p. 83.

L. BIANCHI [20]). Hence the system (2. 1) and its conjugated system (8. 11) are simultaneously completely integrable.

We remark that according to (8. 8) the relation of the systems (2. 1) and (8. 11) is mutually reciprocal [21]).

Collecting results we have:

Theorem IX. *If in the system of equations* (3. 2) *the independent variable* $\zeta = \xi^0$ *is fixed, in the* X_n *obtained from the* X_{n+1} *of the* ξ^α, $\alpha = 0, 1, \ldots, n$, *by reduction with respect to the congruence* $\xi^\varkappa = const.$; $\varkappa = 1, \ldots, n$, *a connexion* $\Gamma^\varkappa_{\mu\lambda}$ *and a tensor* $a_{\mu\lambda} = \Gamma_{\mu\lambda}$ *are determined. If* $a_{\mu\lambda}$ *has the rank n this tensor can be used as fundamental tensor and there exists another connexion* $'\Gamma^\varkappa_{\mu\lambda} = 2\left\{{\varkappa \atop \mu\lambda}\right\} - \Gamma^\varkappa_{\mu\lambda}$. *If in the equations* (2. 1) $\Gamma^\nu_{\mu\lambda}$ *is replaced by* $'\Gamma^\nu_{\mu\lambda}$ *a new system of equations of the second order arises, called the conjugated system of* (2. 1) *after* BIANCHI. *The system* (2. 1) *and its conjugated system* (8. 11) *are reciprocal and simultaneously completely integrable.*

Epe [22]) — *Praha,* April 1949.

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[20]) Cf. [1].

[21]) Cf. [1], V, p. 320.

[22]) The substantial part of this paper was written during my studies by Prof. J. A. SCHOUTEN in Epe, Nederland.

[23]) I read this paper only when my paper had been communicated in Academy. Under the transformation (7. 1) there is the intrinsic connexion $\Gamma^\nu_{\alpha\beta}$ ([8], (2. 2 *b*), p. 206) related with the system (1. 1) which specializes for the system (2. 1) to the connexion (7. 2). The fundamental invariant $P^i_{j\alpha\beta}$ ([8], (2. 8 *b*), p. 207) specializes to the tensor (7. 3).

Mathematics. — *Affinzusammenhängende Mannigfaltigkeiten von Linienelementen, die ein Inhaltsmass besitzen* [1]. Von O. VARGA. (Communicated by Prof. J. A. SCHOUTEN.)

(Communicated at the meeting of September 24, 1949.)

In einem beliebigen Finslerschen Raum, der entsprechend der Cartanschen Theorie [2]) als Linienelementmannigfaltigkeit aufgefasst werden soll, besitzt ein integrables — aus Punkten bestehendes — Gebiet bekanntlich nur in Bezug auf ein Richtungsfeld ein Inhaltsmass. Es gibt aber auch eine solche Klasse von Finslerschen Räumen, für die ein integrables Gebiet ein, von einem Richtungsfeld unabhängiges Inhaltsmass im gewöhnlichen Sinne besitzt. Diese Klasse von Räumen ist dadurch gekennzeichnet, dass für sie eine sogenannte Cartansche Geometrie gilt, d.h. eine Geometrie, die aus euklidisch-zusammenhängenden Hyperflächenelementen besteht [3]).

Hier soll nun untersucht werden, unter welchen Bedingungen eine affinzusammenhängende Linienelementmannigfaltigkeit ein relatives, d.h. von einem Richtungsfeld abhängiges Inhaltsmass besitzt. Ferner sollen die Bedingungen erörtert werden für ein Inhaltsmass im gewöhnlichen Sinne. Es wird sich zeigen, dass in gewissen Spezialfällen von Mannigfaltigkeiten für die ein absoluter — d.h. vom Wege unabhängiger — Parallelismus der Linienelemente besteht, stets ein relatives Inhaltsmass in Bezug auf ein Feld von paralellen Linienelementen existiert. Es werden weiter zwei Klassen von Mannigfaltigkeiten angegeben, in denen ein gewöhnliches Inhaltsmass existiert. Die eine ist wieder eine Unterklasse der Mannigfaltigkeiten mit absolutem Parallelismus der Linienelemente. Für die zweite Klasse hingegen ist diese Bedingung nicht notwendig. Diese Klasse ist das genaue Analogon derjenigen Finslerschen Räume für die ein Inhaltsmass im gewöhnlichen Sinne vorhanden ist. Wird in der Linienelementmannigfaltigkeit eine Finslersche Metrik eingeführt, so müssen natürlich beide Klassen von Mannigfaltigkeiten zusammenfallen oder die erstere eine Unterklasse der zweiten sein. Dies wird am Schluss der Arbeit nachgewiesen. Wir stellen der bequemeren Lesbarkeit halber die wichtigsten Begriffe und Formeln über affinzusammenhängende Linienelementmannigfaltigkeiten so weit wir sie hier benötigen zusammen [4]).

1. *Affinzusammenhängende Mannigfaltigkeiten von Linienelementen.* Sind $\xi^\varkappa$ ($\varkappa = 1, 2, \ldots, n$) die Koordinaten eines Punktes und wird eine

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[1]) Ich spreche Herrn J. A. SCHOUTEN für wertvolle Ratschläge, durch die meine Arbeit verbessert wurde, meinen besten Dank aus.

[2]) E. CARTAN [1], S. 4—5. (Siehe Schriftenverzeichnis am Ende dieser Arbeit.)

[3]) E. CARTAN [2], S. 12—20 und E. CARTAN [1], S. 29—32.

[4]) O. VARGA [1].

Richtung durch diesen Punkt durch das Verhältnis der $v^\varkappa (\varkappa = 1, 2, \ldots, n)$ [5]) charakterisiert, so ist ein Linienelement der $(2n-1)$-dimensionalen Mannigfaltigkeit durch (ξ, v) bestimmt.

Die Mannigfaltigkeit heisst affinzusammenhängend, wenn für sie Grössen $C_{\dot\mu\,\dot\lambda}^{\ \ \dot\varkappa}$, $\Gamma_{\mu\lambda}^{\varkappa}$ [6]) gegeben sind, die bei der Koordinatentransformation

$$\xi^{\varkappa'} = \xi^{\varkappa'}(\xi^\varkappa) \quad \mathrm{Det}\,(A_\varkappa^{\varkappa'}) \neq 0 \ : \ A_\varkappa^{\varkappa'} \overset{\mathrm{def}}{=} \partial_\varkappa \xi^{\varkappa'} \quad . \ . \ . \quad (1.1)$$

$$v^{\varkappa'} = A_\varkappa^{\varkappa'}\, v^\varkappa , \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1.1a)$$

dem Transformationsgesetz

$$C_{\dot\mu'\,\dot\lambda'}^{\ \ \ \dot\varkappa'} = A_{\mu'\lambda'\varkappa}^{\mu\lambda\varkappa'}\, C_{\dot\mu\,\dot\lambda}^{\ \ \dot\varkappa}, \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1.2)$$

$$\Gamma^{*\,\varkappa'}_{\mu'\lambda'} = A^{\mu\lambda\,\varkappa'}_{\mu'\lambda'\varkappa}\, \Gamma^{*\,\varkappa}_{\mu\lambda} + A^{\varkappa'}_{\varrho}\, \partial_{\mu'} A^{\varrho}_{\lambda'} \quad . \ . \ . \ . \ . \quad (1.3)$$

genügen, in den $v^\varkappa$ beziehungsweise von (-1)-ter und nullter Ordnung homogen sind und die Relationen

$$C_{\dot\mu\,\dot\lambda}^{\ \ \dot\varkappa}\, v^\mu = C_{\dot\lambda\,\dot\mu}^{\ \ \dot\varkappa}\, v^\mu = 0 \quad . \ . \ . \ . \ . \ . \quad (1.4)$$

befriedigen [7]). Setzen wir

$$\Lambda_\mu^\varkappa = \Gamma^{*\,\varkappa}_{\mu\lambda}\, v^\lambda \quad [8]), \quad . \ . \ . \ . \ . \ . \quad (1.5)$$

so ist für ein differenzierbares Vektorfeld $u^\varkappa(\xi, v)$ das invariante Differential $\delta u^\varkappa$ durch

$$\delta u^\varkappa = d u^\varkappa + C_{\dot\mu\,\dot\lambda}^{\ \ \dot\varkappa}\, (d v^\mu + \Lambda_\varrho^\mu\, d\xi^\varrho)\, u^\lambda + \Gamma^{*\,\varkappa}_{\mu\lambda}\, u^\lambda\, d\xi^\mu \quad . \ . \quad (1.6)$$

bestimmt.

Von den drei Krümmungsaffinoren benötigen wir hier bloss den Affinor

$$\left.\begin{aligned} P_{\nu\,\dot\mu\,\dot\lambda}^{\ \ \ \dot\varkappa} &= 2\,\partial_{[\nu}\,\Gamma^{*\,\varkappa}_{\mu]\lambda} - 2\,\Lambda^\varrho_{[\nu}\,\frac{\partial}{\partial v^{|\varrho|}}\,\Gamma^{*\,\varkappa}_{\mu]\lambda} + \\ &+ 2\,C_{\dot\varrho\,\dot\lambda}^{\ \ \dot\varkappa}\,(\partial_{[\nu}\,\Lambda^\varrho_{\mu]} + \Lambda^\varrho_{[\nu\,|\tau|}\,\Lambda^\tau_{\mu]}) + 2\,\Gamma^{*\,\varrho}_{[\mu\,|\lambda|}\,\Gamma^{*\,\varkappa}_{\nu]\,\varrho}\,, \quad \Lambda^\varrho_{\nu\tau} \overset{\mathrm{def}}{=} \frac{\partial}{\partial v^\tau}\,\Lambda^\varrho_\nu \quad [9]). \end{aligned}\right\} \quad (1.7)$$

Aus ihm lässt sich durch Überschiebung mit v^λ der Affinor

$$P_{\nu\,\dot\mu}^{\ \ \ \dot\varkappa} \overset{\mathrm{def}}{=} P_{\nu\,\dot\mu\,\dot\lambda}^{\ \ \ \dot\varkappa}\, v^\lambda = 2\,\partial_{[\nu}\,\Lambda^\varkappa_{\mu]} + 2\,\Lambda^\varkappa_{[\nu\,|\varrho|}\,\Lambda^\varrho_{\mu]} \quad . \ . \ . \quad (1.8)$$

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[5]) Da in vorliegender Arbeit die SCHOUTENsche Kernindexmethode benützt wird, erwies es sich als notwendig, einige Änderungen gegenüber Bezeichnungen früherer Arbeiten des Verf. zu machen. Auf diese Änderungen soll stets an entsprechender Stelle hingewiesen werden. Wir verwenden hier $v^\varkappa$ statt $x'^\varkappa$ in O. VARGA [1].

[6]) $C_{\dot\mu\,\dot\lambda}^{\ \ \dot\varkappa}$ und $\Gamma^{*\,\varkappa}_{\mu\lambda}$ statt $C^\varkappa_{\lambda\mu}$ und $\Gamma^{*\,\varkappa}_{\lambda\mu}$ in O. VARGA [1].

[7]) Diese Definition der affinzusammenhängenden Mannigfaltigkeit in O. VARGA [1], Paragraf 1.

[8]) $\Lambda^\varkappa_\mu$ statt $G^\varkappa_\mu$ in O. VARGA [1].

[9]) $P_{\nu\,\dot\mu\,\dot\lambda}^{\ \ \ \dot\varkappa}$ statt $P^\varkappa_{\lambda.\mu\nu}$ in O. VARGA [1].

herleiten [10]). Schliesslich erwähnen wir noch den aus (1.7) und (1.8) ableitbaren Hauptkrümmungsaffinor

$$T^{\cdot}_{\nu\,\cdot\,\mu\,\cdot\,\lambda}{}^{\varkappa} = P^{\cdot}_{\nu\,\cdot\,\mu\,\cdot\,\lambda}{}^{\varkappa} - P^{\cdot}_{\nu\,\cdot\,\mu}{}^{\varrho}\, C^{\cdot}_{\varrho\,\cdot\,\lambda}{}^{\varkappa} \quad [11]). \quad . \quad . \quad . \quad . \quad (1.9)$$

2. *Definition des Inhaltsmasses und Bestimmung der entsprechenden Dichten.*

In der affinzusammenhängenden Mannigfaltigkeit muss jede Grösse geometrischer Natur lediglich von den, den Zusammenhang bestimmenden, Grössen (1.2), (1.3) und deren Ableitungen hinsichtlich den $\xi^{\varkappa}$ bzw. $v^{\varkappa}$ abhängen.

Wir zeichnen nun in einem $(2n-1)$-dimensionalen Teilbereich der Mannigfaltigkeit ein Richtungsfeld

$$v^{\varkappa} = t^{\varkappa}(\xi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

aus und bezeichnen das zugeordnete, als n-dimensional und integrabel vorausgesetzte, Punktgebiet mit P. Ist dann $\mathfrak{q}$ eine Dichte vom Gewicht $+1$, die von $\xi^{\varkappa}, v^{\varkappa}, C^{\cdot}_{\mu\,\cdot\,\lambda}{}^{\varkappa}, \Gamma^{*}_{\mu\,\cdot\,\lambda}{}^{\varkappa}$ und den Ableitungen erster Ordnung der beiden letzten Grössen hinsichtlich $\xi^{\varkappa}$ und $v^{\varkappa}$ abhängt, so definieren wir das Inhaltsmass in Bezug auf das Richtungsfeld (2.1) durch

$$\int_{P} \ldots \int \mathfrak{q}\, d\xi^1 . d\xi^2 \ldots d\xi^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

Aus dem Transformationsgesetz

$$\overset{(\varkappa')}{\mathfrak{q}} = \Delta^{-1}\, \overset{(\varkappa)}{\mathfrak{q}} \quad \Delta \overset{\text{def}}{=} \text{Det}\,(A^{\varkappa'}_{\varkappa}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

folgt

$$\partial_{\mu} \log \overset{(\varkappa)}{\mathfrak{q}} = A^{\mu'}_{\mu}\, \partial_{\mu'} \log \overset{(\varkappa')}{\mathfrak{q}} + \partial_{\mu} \log \Delta \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

Anderseits folgt aus dem Transformationsgesetz (1.3) der $\Gamma^{*}_{\mu\,\cdot\,\lambda}{}^{\varkappa}$ wenn

$$\Gamma^{*}_{\mu} \overset{\text{def}}{=} \Gamma^{*}_{\mu\,\cdot\,\varkappa}{}^{\varkappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.5)$$

gesetzt wird:

$$\Gamma^{*}_{\mu} = A^{\mu'}_{\mu}\, \Gamma^{*}_{\mu'} + \partial_{\mu} \log \Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.6)$$

Subtraktion von (2.4) und (2.6) zeigt, dass

$$w_{\mu} \overset{\text{def}}{=} \partial_{\mu} \log \overset{(\varkappa)}{\mathfrak{q}} - \Gamma^{*}_{\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.7)$$

ein kovariantes Vektorfeld festlegt. Wenn nun ein Vektorfeld w_{μ} von der Art vorgegeben wird, dass (2.7) ein integrables System für $\mathfrak{q}$ darstellt,

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[10]) $P^{\cdot}_{\nu\,\cdot\,\mu\,\cdot\,\lambda}{}^{\varkappa}$ statt $P_{\lambda\,\cdot\,\mu\,\nu}{}^{\varkappa}$ in O. VARGA [1].

[11]) Dieser für die Äquivalenz von affinzusammenhängenden Mannigfatligkeit grundlegende Affinor wurde in O. VARGA [1], § 3 eingeführt.

dann ist q eine invariante Dichte. Die Bedingungen der vollständigen Integrabilität sind

$$\partial_{[\nu}\, \Gamma^*_{\mu]} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho}\, \partial_{\nu]}\, r^\varrho + \partial_{[\nu}\, w_{\mu]} = 0, \quad . \quad . \quad . \quad . \quad (2.8)$$

wobei beachtet wurde, dass die Γ^*_μ vermöge (2.1) Funktionen der ξ^x sind. Die Gleichungen (2.8) bilden für die w_μ ein vollständig integrierbares System, was bei Benutzung dieser Gleichungen selbst unmittelbar folgt. Eine Lösung w_μ von (2.8) führt aber zu einer Dichte q die im allgemeinen nicht den an das Inhaltsmass gestellten Forderungen genügt, da q auch von den Ableitungen der r^ϱ abhängen wird.

Wir gehen daher so vor, dass wir w_λ als Funktion der ξ^x und r^x wählen und dann (2.8) als ein Differentialgleichungssystem für r^x betrachten. Kann ein w_λ so angegeben werden, dass (2.8) eine Lösung r^x besitzt, dann ist q eine Dichte und es ist ein Inhaltsmass in Bezug auf das Richtungsfeld r^x bestimmt.

Die Gleichungen (2.8) wollen wir noch umformen. Verjüngung des Hauptkrümmungsaffinors (1.8) gibt wegen (1.7)

$$T^{\cdot\cdot\lambda}_{\nu\,\mu\,\lambda} = 2\,\partial_{[\nu}\, \Gamma^*_{\mu]} - 2\,\frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho}\, \Lambda^\varrho_{\nu]}. \quad . \quad . \quad . \quad . \quad . \quad (2.9)$$

Beachten wir ferner, dass w_x Funktion der ξ^x und r^x ist, so nimmt (2.8) die Form

$$\tfrac{1}{2}\, T^{\cdot\cdot\lambda}_{\nu\,\mu\,\lambda} + \partial_{[\nu}\, w_{\mu]} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho}\, \Lambda^\varrho_{\nu]} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho}\, \partial_{\nu]}\, r^\varrho + \frac{\partial w_{[\mu}}{\partial v^\varrho}\, \partial_{\nu]}\, r^\varrho = 0 \quad [12]) \quad (2.10)$$

an.

Für den Fall in dem ein Inhaltsmass im gewöhnlichen Sinne vorhanden ist, muss $w_\mu + \Gamma^*_\mu$ von dem Richtungsfeld v^x unabhängig sein und es muss also die Gleichung

$$\frac{\partial w_\mu}{\partial v_x} + \frac{\partial \Gamma^*_\mu}{\partial v^x} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.11)$$

gelten. Man erhält dann statt (2.10) das System

$$\tfrac{1}{2}\, \Gamma^{\cdot\cdot\lambda}_{\nu\,\mu\,\lambda} + \partial_{[\nu}\, w_{\mu]} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho}\, \Lambda^\varrho_{\nu]} = 0. \quad . \quad . \quad . \quad . \quad (2.12)$$

Eine aus (2.11) und (2.12) gewonnene Lösung w_μ führt dann auf Grund von (2.7) zu einer, von einem Richtungsfeld unabhängigen, Dichte q.

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[12]) Formel (2.10) und ihre Herleitung verdanke ich einer brieflichen Mitteilung des Herrn J. A. SCHOUTEN.

3. *Kennzeichnung von Mannigfaltigkeiten mit relativen und gewöhnlichem Inhaltsmass und Zusammenhang der letzteren mit den Finslerschen Räumen.*

Wenn wir das Vektorfeld

$$w_\mu = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.a)$$

wählen, so reduziert sich (2.10) auf

$$\tfrac{1}{2} T^{\cdot}_{\nu\,\mu\,\lambda}{}^{\lambda} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho} \Lambda^\varrho_{\nu]} + \frac{\partial \Gamma^*_{[\mu}}{\partial v^\varrho} \partial_{\nu]} r^\varrho = 0 . \quad . \quad . \quad . \quad . \quad (3.1)$$

Dieser Spezialfall ist wie (2.7) zeigt durch die Forderung ausgezeichnet dass Γ^*_μ die Ableitung des Logarithmus einer Dichte vom Gewicht $+1$ sein soll. Die Gleichungen (3.1) sind sicher erfüllt für den besonderen Fall dass

$$T^{\cdot}_{\nu\,\mu\,\lambda}{}^{\lambda} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

und

$$\partial_\nu r^\varrho = - \Lambda^\varrho_\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.3)$$

gewählt wird. Soll das System (3.3) die r^ϱ nach Angabe eines willkürlichen Linienelementes $\underset{0}{\xi^\varkappa}\ \underset{0}{r^\varkappa}$ als Anfangsbedingung, eindeutig als Ortsfunktion der $\xi^\varkappa$ bestimmen, so muss (3.3) ein unbeschränkt integrables System sein. Die notwendigen und hinreichenden Bedingungen dafür sind aber, wie eine einfache Rechnung zeigt, wegen (2.8):

$$P^{\cdot}_{\nu\,\mu}{}^{\varkappa} = 0 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.4)$$

Die Bedingungen (3.2) und (3.4) sind wegen (1.9) auch mit (3.4) und

$$P^{\cdot}_{\nu\,\mu\,\varkappa}{}^{\varkappa} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

gleichbedeutend. Die Parallelübertragung des Linienelementes (ξ, v) von $\xi^\varkappa$ zum willkürlichen Nachbarpunkt $\xi^\varkappa + d\xi^\varkappa$ ist durch

$$dv^\varkappa + \Lambda^\varkappa_\varrho \, d\xi^\varrho = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.6)$$

definiert[13]). Diese Gleichungen sind aber mit (3.3) identisch. Daher existiert in einer Mannigfaltigkeit für die (3.4) gilt ein absoluter Parallelismus für die Linienelemente. Die Gleichungen (2.7) und (3, a) ergeben für das Inhaltsmass die Formel

$$I = \int_P \ldots \int C^{\int_{Q_0}^{Q} \Gamma^*_\varrho \, d\tau^\varrho} \, d\xi^1 \ldots d\xi^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.7)$$

in der Q_0 einen festgewählten und Q einen veränderlichen Punkt von P bedeutet. Wir können unser Ergebnis zusammenfassen in dem

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[13]) O. VARGA [1] Formeln (2.3) und (2.4).

Satz 1. *In der affinzusammenhängenden Linienelementmannigfaltigkeit existiert sicher ein relatives Inhaltsmass in Bezug auf ein Richtungsfeld, wenn für ihre Linienelemente ein absoluter Parallelismus besteht (d.h.* (3.4) *gilt), ferner* (3.5) *besteht und man das Richtungsfeld mit einem beliebigen Feld von paralellen Linienelementen zusammenfallen lässt.*

Wir wollen nun Mannigfaltigkeiten kennzeichnen in denen ein gewöhnliches — d.h. von einem Richtungsfeld unabhängiges — Inhaltsmass vorhanden ist. Fordern wir wiederum speziell, dass Γ_μ^* die Ableitung des Logarithmus einer Dichte vom Gewicht $+1$ ist, dann bestehen zunächst die Gleichungen (3.a) und daher muss wegen (2.11) die Beziehung

$$\frac{\partial \Gamma_\mu^*}{\partial v^\varrho} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.8)$$

gelten. Daraus folgen dann wegen (2.12) die Relationen (3.2). Die letzteren Relationen (d.h. das Verschwinden des verjüngten Hauptkrümmungsaffinors) bedeuten nun wegen (1.9), dass entweder (3.5) und

$$C_\mu \overset{\text{def}}{=} C_{\mu\,\cdot\,\cdot}^{\cdot\,\cdot\,\varrho} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.9)$$

gelten, oder dass (3.5) und (3.4) gelten. Dies gibt den

Satz 2. *Affinzusammenhängende Mannigfaltigkeiten von Linienelementen die entweder durch* (3.4), (3.5) *und* (3.8), *oder durch* (3.5), (3.8) *und* (3.9) *gekennzeichnet sind, besitzen stets ein gewöhnliches Inhaltsmass.*

Wir zeigen nun, dass falls der Mannigfaltigkeit eine Finslersche Metrik aufgeprägt ist, von den beiden in Satze 2 vorkommenden Mannigfaltigkeiten die durch (3.4), (3.5) und (3.8) charakterisierte ein Spezialfall der durch (3.5), (3.8) und (3.9) charakterisierten Mannigfaltigkeit ist. Bei Einführung einer Finslerschen Metrik kann man den Krümmungsaffinor (1.7) (der dann mit lateinischem Kernbuchstaben bezeichnet werden soll) in der Form

$$R_{\nu\,\cdot\,\cdot\,\lambda}^{\cdot\,\mu\,\cdot\,\varkappa}\, g^{\varkappa\varrho}\, R_{\nu\mu\lambda\varrho} \quad [14]) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.10)$$

darstellen. Da der Affinor auf der rechten Seite in den beiden letzten Indizes schiefsymmetrisch ist [15]), folgt daraus, dass (3.5) von selbst erfüllt ist. Aus Gleichungen (3.9) folgen ferner im Finslerschen Raum die Gleichungen (3.8) [16]). Damit ist zunächst gezeigt, dass bei Einführung einer Finslerschen Metrik, die durch (3.5), (3.8) und (3.9) gekennzeichnete Mannigfaltigkeit allein durch (3.9) charakterisiert werden kann.

Wir weisen nun nach, dass im Finslerschen Raum die Beziehungen

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[14]) E. CARTAN [1] S. 33.
[15]) E. CARTAN [1] S. 36.
[16]) E. CARTAN [1] S. 35 Formel (44).

(3. 4) und (3. 8) die Beziehungen (3. 9) nach sich ziehen. Da (2. 7) und (3. *a*) gilt, folgt wegen unserer Voraussetzungen

$$\frac{\partial \log \mathfrak{q}}{\partial \xi^\varrho} = \Gamma_\varrho^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.11)$$

Im Finslerschen Raum gilt aber für den Fundamentaltensor $g_{\lambda\mu}$ genau wie im Riemannschen Raum

$$\frac{\partial g_{\lambda\mu}}{\partial \xi^\varrho} = \Gamma_{\lambda\mu\varrho} + \Gamma_{\mu\lambda\varrho} \,, \quad ^{17}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.12)$$

woraus für die Determinante $\mathfrak{g}$ der $g_{\lambda\mu}$

$$\partial_\varrho \log \sqrt{\mathfrak{g}} = \Gamma_\varrho \,, \quad \Gamma_\varrho \overset{\text{def}}{=} \Gamma^\mu_{\varrho\mu} \quad . \quad . \quad . \quad . \quad . \quad (3.13)$$

folgt.

Aus (3. 11) und (3. 13) erhält man

$$\Gamma_\varrho^* - \Gamma_\varrho = \partial_\varrho \log \frac{\mathfrak{q}}{\sqrt{\mathfrak{g}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.14)$$

Da $\mathfrak{q}$ und $\sqrt{\mathfrak{g}}$ beide Dichten vom Gewicht $+1$ sind, ist die Differenz auf der linken Seite von (3. 14) ein kovarianter Vektor. Für diese Differenz gilt nun anderseits

$$\Gamma_\varrho^* - \Gamma_\varrho = C_\varkappa \, G_\varrho^\varkappa \,. \quad ^{18}) \quad . \quad . \quad . \quad . \quad . \quad (3.15)$$

Da die $G_\varrho^\varkappa$ keinen Affinor bestimmen, kann (3. 15) nur bestehen falls

$$C_\varkappa \, G_\varrho^\varkappa = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.16)$$

ist. Da nun

$$G_\varrho^\varkappa \, v^\varrho = G^\varkappa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.17)$$

ist [19]), wo die $G^\varkappa$ von Null verschieden sind, kann die Determinante der $G_\varrho^\varkappa$ nicht verschwinden. Daraus folgt aber

$$C_\varrho = 0. \qquad \text{w. z. b. w.}$$

SCHRIFTENVERZEICHNIS.

E. CARTAN, [1] Les espaces de Finsler, Actualités scientifiques et industrielles, **79** (Paris, 1934).

[2] Les espaces de Finsler, Actualités scientifiques et industrielles, **72** (Paris, 1933).

O. VARGA, [1] Über affinzusammenhängende Mannigfaltigkeiten von Linienelemente insbesondere deren Aquivalenz. Publicationes Mathematicate, Debrecen. Vol. 1 (1949), 7—17.

Mathematisches Seminar der Universität Debrecen (Ungarn.)

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[17]) E. CARTAN [1] S. 5 Formel (7).

[18]) E. CARTAN [1] S. 14 Formel (12) und S. 16 Formel X.

[19]) E. CARTAN [1] S. 16 Formel 20.

Botany. — *On the mechanism of auxin action.* I. *The influence of indole-3-acetic acid on the respiration of Saccharomyces cerevisiae.* By L. ANKER. (Communicated by Prof. V. J. KONINGSBERGER.)

(Communicated at the meeting of June 25, 1949.)

Introduction.

In a recent publication called "The mechanism of auxin action" AUDUS (1949) reviewed critically the suggestions put forward in the last decade on the way in which growth substances affect the rate of cell elongation.

According to one of these suggestions, originated by BONNER (1933) and supported by COMMONER and THIMANN (1941) and others, the primary action of growth substances consists in stimulation of the aerobic respiration.

Promotion of the rate of oxygen uptake, however, could not be confirmed in all cases. VAN HULSSEN (1936, in *Avena*-coleoptiles), BURSTRÖM (1942, in wheat roots) and others showed, that respiration was not altered by adding indole-3-acetic acid.

In the present investigation the influence of indole-3-acetic acid (hetero-auxin) on the rate of oxygen uptake of *Saccharomyces cerevisiae* in pure culture is studied.

The experimental part is divided in four sections:

a. experiments in which hetero-auxin did not influence aerobic respiration;

b. experiments in which acceleration of the oxygen uptake was observed;

c. effect of various concentrations of hetero-auxin;

d. microscopical examination of yeast cells treated with hetero-auxin.

From these experiments it is concluded, that hetero-auxin promotes respiration in an indirect way, namely by stimulating the breakdown of reserve carbohydrates into respirable substrate.

Material and methods.

Cells of *Saccharomyces cerevisiae*, inoculated in a dilute malt extract, were cultivated in continuously shaken flasks during two days at a constant temperature of 25° C. After centrifuging and washing a suspension was made in a $2\frac{1}{2}$ % KH_2PO_4 solution (pH = 4.5), two ml. of which were pipetted into Warburg vessels in order to determine manometrically the rate of oxygen uptake by endogenous respiration (consumption of reserve food). Next, after about three quarters of an hour, hetero-auxin and — if necessary — glucose were added (end volume of the medium: $2\frac{1}{2}$ ml).

The subsequent readings were divided by the initial value of endogenous respiration. This ratio was plotted on the graph.

The experiments were carried out at 25° C.

Experimental part.

a. *The effect of hetero-auxin on exogenous respiration, the substrate occurring in excess.*

These experiments were made in order to study whether hetero-auxin affects respiration if glucose is added in concentrations at which the enzyme system is saturated.

When for this purpose glucose was present in concentrations ranging from 3 to 0.5 per cent no significant effect of hetero-auxin (10 mg/l), in any case no promotion, could be observed. Fig. 1 shows the results with 1.0, 0.75 and 0.50 % glucose.

In accordance with data of GEIGER–HUBER (1934) a sugar concentration

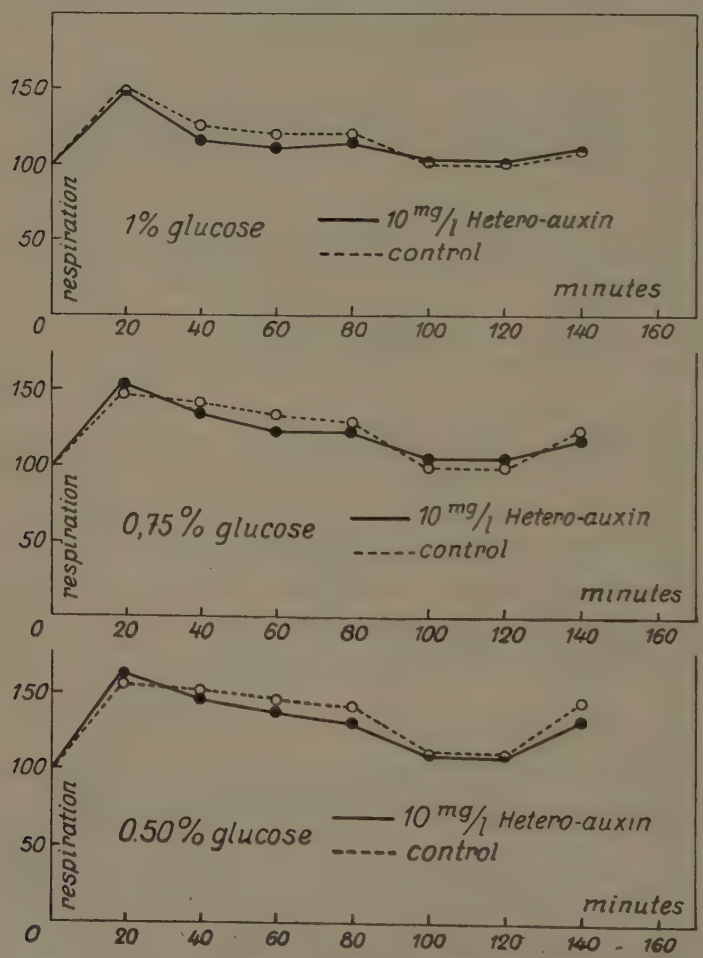

Fig. 1. No promoting influence of hetero-auxin on respiration, if substrate-concentration is supra-optimal.

of about 0.2 per cent was found to be optimal concerning the respiration of yeast cells. In lower concentrations glucose proved to become limiting factor in the process.

From the above mentioned experiments, in which substrate was abundant, one may conclude, *that hetero-auxin does not increase the respiration capacity.*

b. *The effect of hetero-auxin when substrate concentration is the limiting factor in respiration.*

When, however, glucose was added in suboptimal concentrations a marked promoting effect of hetero-auxin on respiration appeared immediately after the beginning of the experiment (see fig. 2, glucose concentration: 0.125 %). At the moment when the rate of oxygen uptake

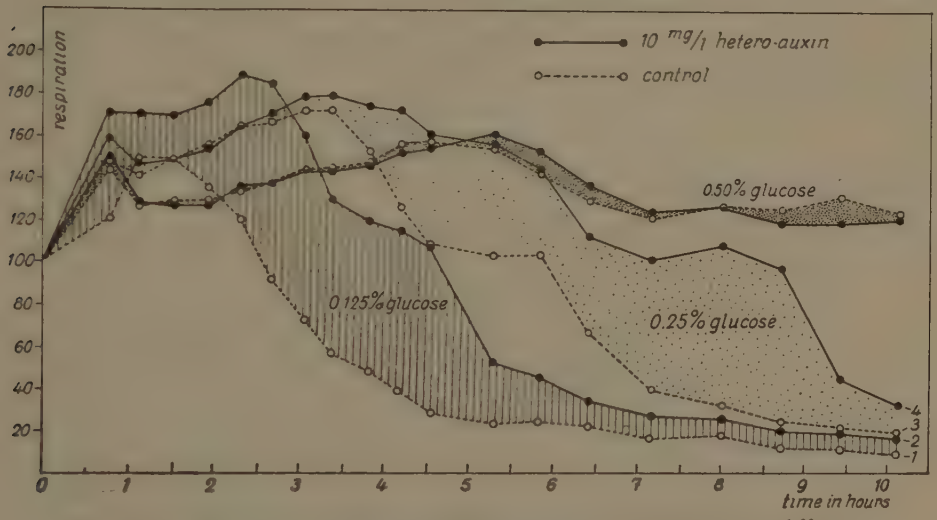

Fig. 2. The promoting influence of hetero-auxin on respiration at different substrate-concentrations.

in the control started to decrease (after about 1½ hour), the respiration of the treated cells (10 mg/l hetero-auxin) was still increasing. In the next 4 hours of the experiment the difference between treated cells and controls became maximal.

The same effect was obtained at a double glucose concentration (0.25 %) but only after the moment at which substrate became limiting factor (fig. 2, after about 3 hours). In the first hours, when this concentration was still supraoptimal, no promotion of respiration by 10 mg/l hetero-auxin was found. The course of these curves will be dealt with in the discussion.

In this experiment, as well as in that of fig. 1, no significant influence was shown at the 0.50 % glucose concentration.

A strongly promoting effect of hetero-auxin (10 mg/l) was observed in experiments without glucose (fig. 3).

These results show, *that hetero-auxin is able to promote the respiration of glycogen containing yeast cells when the amount of added substrate is the limiting factor in the process.*

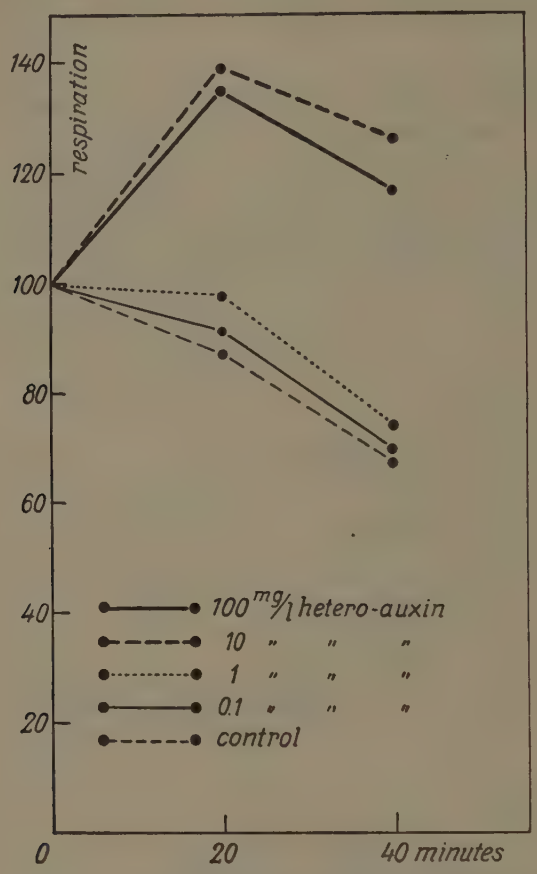

Fig. 3. Influence of hetero-auxin in four different concentrations on endogenous respiration.

c. *Effect of various concentrations of hetero-auxin.*

The results of an experiment concerning the question which concentration of hetero-auxin causes maximal stimulation of respiration is represented in fig. 4.

During the first hour of this experiment, in which glucose was added in a very low concentration (0.03 %), hetero-auxin in a concentration of 10 mg/l seemed to function optimally.

A similar experiment, whithout addition of glucose, confirmed this result (fig. 3).

In the subsequent part of the experiment of fig. 4, however, the influence of the concentration of 10 mg/l vanished gradually, whereas the stimulation by the highest concentration remained constant.

Hardly any influence of the lowest concentration of hetero-auxin used (0.1 mg/l) could be observed.

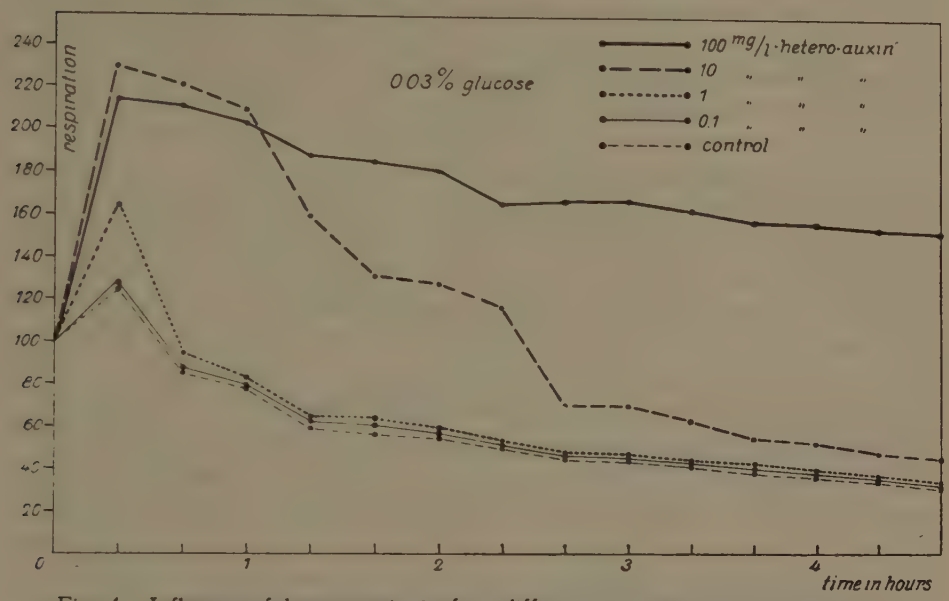

Fig. 4. Influence of hetero-auxin in four different concentrations on respiration (at the start of the experiment 0.03 % glucose was added).

A possible explanation of the existence of an optimal hetero-auxin concentration will be presented in the discussion.

d. *Microscopical observation of yeast cells treated with hetero-auxin.*

Glycogen containing yeast cells were suspended in a buffered solution (pH: 4.5) of hetero-auxin (100 mg/l) without substrate. Microscopical examination on the next days showed, that glycogen disappeared much sooner from the hetero-auxin treated cells than from the untreated ones. On the second day, after staining with jodine, the treated ones turned yellowish whereas the controls still coloured brown. In other cases a marked difference was apparent on the first day after inoculation.

This result seems to indicate *that hetero-auxin accelerates reserve carbohydrate mobilisation in yeast.*

Discussion.

From these results it may be concluded, that the promoting influence of hetero-auxin on the oxygen uptake of *Saccharomyces cerevisiae* is not the primary effect of this growth substance. Only when respiration is dependent on the presence of reserve substances the effect can be observed. Hence it is suggested, that the hetero-auxin induced increase of the rate of respiration is caused by an acceleration of the glycogen mobilisation.

This view is supported by REINDERS (1938), BAUSOR (1942), MITCHELL and BROWN (1945), RASMUSSEN (1947), SMITH (1948) and others who

showed a rapid increase of the amount of reducing sugars at the expense of starch present in the cell after application of growth substances.

From such observations, however, it is not permitted to conclude, that improved diastase- or phosphorylase activity is to be visualized as the primary effect of growth substances. WEINTRAUB and WOOD (1947) and the author (unpublished experiments) failed to demonstrate a direct stimulation of purified diastase by hetero-auxin in vitro. SMITH, LANGELAND and STOTZ (1947) did not succeed in confirming EYSTER's results (1943, 1946), who showed a releasing of diastase from the bound state on charcoal by growth substances. These negative results might indicate, that improved hydrolysis (or phosphorolysis) of reserve carbohydrates is effected by a more fundamental process.

According to the opinion of PFLÜGER (1905), PRZYLECKI (1934), WILLSTÄTTER and RHODEWALD (1934) glycogen in the cell in some way is either bound to- or surrounded by proteins. TSAI (1937) and MEYER and PRESS (1941) showed, that glycogen, present in extracts of organs is attacked by α- and β-amylase in vitro only when the albumin had been precipitated. NORTHEN (1942) ascribed the decrease in structural viscosity of protoplasm, caused by hetero-auxin in his experiments to dissociation of cellular proteins. Thus, an influence of hetero-auxin on physicochemical properties of protoplasm, resulting in an unlocking of glycogen from its bound state might be the cause of increased phosphorolysis.

The occurrence of an optimal hetero-auxin concentration in stimulating respiration might be explained by assuming, that higher concentrations are injurious to the cell. Remembering, however, that hetero-auxin (concentration 10 mg/l) just like 0.2 % glucose (optimal glucose concentration) is able to bring about the maximal oxygen uptake, one might imagine that a tenfold of this concentration releases so much substrate that its concentration becomes supraoptimal. In this way the hetero-auxin optimum might be explained as a consequence of the existence of a substrate optimum.

The remarkable course of the curves 3 and 4 of fig. 2, namely the descent in two steps separated by a nearly horizontal stretch, might be explained by assuming a temporary (re)synthesis of glycogen from glucose-1-phosphate or from the added glucose. In the first descent the amount of free substrate becomes the limiting factor; next, the new formed glycogen is used (horizontal part) at a rate equalling that of endogenous respiration at the beginning of the experiment (level 100); lastly the amount of free glycogen becomes the limiting factor causing a decrease of the rate of oxygen uptake to that of the use of bound glycogen.

The experiments of fig. 2 and fig. 4 show, that the duration of hetero-auxin action is limited. This substance is either used or inactivated. The highest concentration of hetero-auxin used (100 mg/l) proved to stimulate respiration during a considerable longer period than a concentration of 10 mg/l did (fig. 4).

Part of the contradictory results reported in literature on stimulation of respiration by growth substances possibly may be ascribed to the different organisms or tissues used. From experiments, which show an increased respiration upon addition of hetero-auxin one is not allowed to conclude to a stimulation of the respiratory enzymes unless substrate is in excess at the moment of adding. In case of a negative response of respiration one is not permitted to draw the conclusion, that growth substances do not stimulate the enzymes unless it is demonstrated, that substrate occurs sufficiently at the moment of adding this substance. From the present results it may be obvious, that in case of limitation of respiration by substrate concentration the amount of reserve carbohydrates in the cell determines whether the results will be positive or negative.

Summary.

Endogenous respiration of starving glycogen containing yeast cells is stimulated by hetero-auxin. Since exogenous respiration could not be stimulated by this growth substance it is stated that this stimulation results from an increased glycogen mobilisation, a process, which supplies substrate to the endogenous respiration.

About 10 mg/l hetero-auxin causes maximal stimulation.

Microscopical observation showed increased glycogen mobilisation on addition of hetero-auxin (100 mg/l) to the medium.

The experiments are continued.

The author wishes to express his gratitude to Prof. Dr. V. J. KONINGSBERGER, Dr. H. P. BOTTELIER and Dr. J. B. THOMAS for their constant interest and valuable criticism.

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Anatomy. — *The Anatomy of Lapemis Hardwickei gray.* I. By R. A. M. BERGMAN. (Department of Anatomy, Batavia.) (Communicated by Prof. J. BOEKE.) [1])

(Communicated at the meeting of June 25, 1949.)

Species. Name.

This short and broad sea-snake has been described by STEJNEGER, TAYLOR and DE ROOY as belonging to a single species, WALL and SMITH distinguish between two species, on account of the number of bands and the form of the ventral shields. The snake is commonly called ular lempe in East Java, ular = snake, lempe seems to be used sometimes in the sense of tired. I spent some time in trying to find the meaning of the scientific name "*Lapemis*" until I found the answer of the riddle in the book of STEJNEGER, who quotes it as an anagram of pelamis, so it is one of the relatively few cases where a biological name has no sense whatever.

At first sight, the most impressive anatomical details are the short and stout build and the dirty yellow colour with the darker brown triangular markings pointing to the belly. I definitely got the impression that all the specimens caught in this area belonged to the one species *Lapemis Hardwickei,* the material however and the lists of scale counts are lost trough the japanese plunder so in this case it is not possible to test this impression by the use of the necessary figures.

Distribution.

Near the coast at Surabaia, it is a very common snake and easily collected. In marked contrast to this experience I have never been able to obtain even a single specimen in Batavia. In this place the native fishermen are well used to bring any strange thing they may catch to the marine aquarium at Pasar Ikan as it will bring them a small reward. Before the war the price offered for a sea-snake was equivalent to a months wages, but no one came. This is not because the people in West Java know the danger of these animals better or are more afraid of poisonous snakes, because they bring plenty of other dangerous snakes, which they know very well, to the laboratory now (1947—1948).

Poison.

In Surabaia, from 1936 to 1941 a malayan fisherman brought practically every week the amount of seasnakes he was asked for. He went out in his prahu at night to catch the snakes as they were easily attracted by the ray of a flashlight. This man could very well distinguish the different species and was not afraid of handling the animals: he knew that, when

[1]) For the tables and lists see part II.

he picked them by the tail and took them quickly out of his bamboo basket, they would just hang head down, unable to redress themselves nor to direct their bite, should they want to do so.

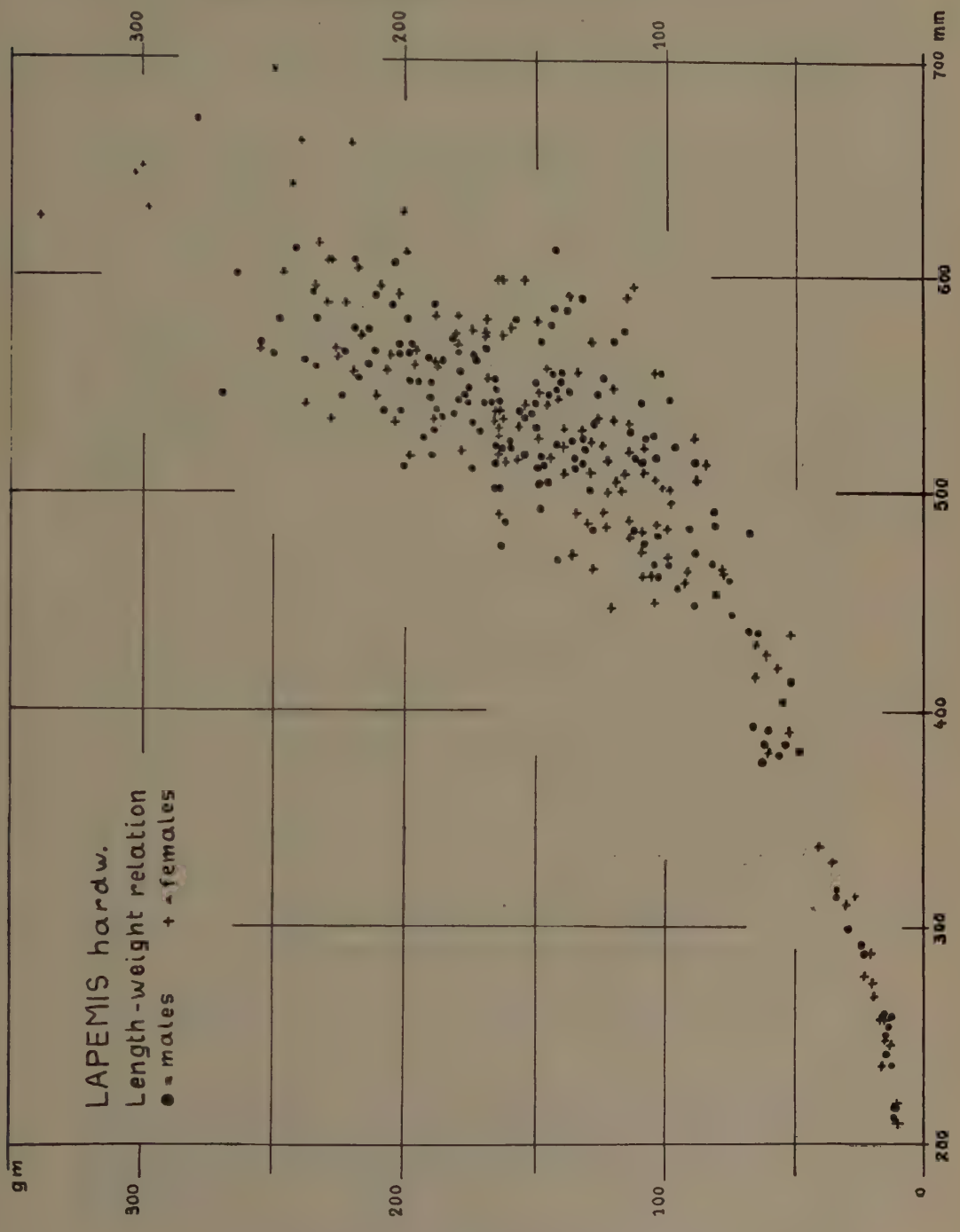


Fig. 1.

Once however he failed to turn up at the usual time and little more than a week later he told us that when he was packing the snakes, one of the lapemis fell besides the basket and bit him on the back of his bare foot. The bite was not very painful, but he had soon felt very tired and a bad headache started. He had slept rather heavily for a few days and recovered gradually without having taken any medicine. He looked rather pale, his foot was slightly swollen and two small puncture points could still be seen.

The snakes are not agressive, I have often tried to make them bite one another, but this was seldom possible and it never gave a fatal result. Once one got accidentally caught with the tail between the side of the aquarium and the lid, it started to screw swiftly round in the water, biting right and left. On another occasion a mouse bitten in the leg did at first not show any symptoms. After I had closed the mouth of the snake over the leg of the mouse and massaged the venom gland, the mouse did not try to run away but only sat down. After a couple of minutes it started to wash its snout with its fore paws and another few minutes later it let itself down and died without any sign of pain, convulsions or being short of breath. The heart was distended and full of dark blood. Lungs and intestines were normal.

Food.

As stated by MALCOLM SMITH, these animals do not thrive in captivity. If kept in salt water they survive easily for a month or six weeks, but finally always die of inanition. They refuse to take food and when fed forcibly, which has to be done gently, they disgorge the fish as soon as they are released. This can be prevented if a moderately large fish is fed and then a piece of string loosely wound around the body cranially from the stomach. But then the snakes regularly strangle themselves by pressing the prey so hard in the string that the ventral aorta becomes completely blocked. During the day they can be stirred up to interrupt this process or the string can be loosened or the prey massaged back into the stomach, but mostly they succeed overnight in their suicidal attempts and are found dead on the following morning. Feeding fish paste through a glass tube brought into the stomach gave no good results either, because this food also was slowly but very regularly regurgitated.

When caught, the stomach is nearly always empty. In 4 individuals out of 552 the stomach was distended, the content being twice a small fish, once a granular mass and once a gelatinous substance.

Sex.

In the adult animals the males have generally stronger tubercles on the scales, but this is by no means an absolute characteristic, it is always difficult and very often impossible to tell the sex of an animal without having seen the ovaries or the testicles. Sometimes the place of the hemipenis is marked by a slight lateral groove on the tail.

WALL gives for Lapemis curtus a sex ratio of 21 males and 28 females. In two consecutive series in Surabaia and in a group of young animals I found the figures listed in the following table. The percentages of male animals are given with their standard deviation; the differences are clearly not significant. For the samples of WALL and the adult group from Surabaia $chi^2 = 0,36$ and P is greater than 0,1 so there is no significant difference between the figures.

	N	Fem	Male	% Male
Wall	49	28	21	43,– ± 7,–
Surabaia	326	165	161	49,4 ± 2,8
Surabaia	139	80	59	42,5 ± 4,5
Sur. young	87	43	44	50,7 ± 5,4

Fertility.

In 12 pregnant snakes WALL found 22 embryos, the young varying in length from 330 to 335 mm, the adults from 686 to 825 mm. In the Surabaia series out of 210 adult females controlled by dissection, 32 were pregnant. Four of these caught in June had just ovulated and for a short while the yolk mass is not clearly divided in a certain number of eggs. In six animals caught in December and January there was a half resorbed egg or an embryo dead before birth. For these 32 snakes the total number of eggs, including all, was 60. Normal embryos were found in 22 animals, 27 in the right uterine tube and 15 in the left one, an average of two young per animal, and an assymetry of 3/2 for right and left.

In the same series, next to the 210 adult females and the 42 embryos, there were 63 young animals belonging to the production of the same year. If we do not count the unknown mortality of the newborn animals, this would mean 105 young in the group, or 53 pregnancies to 210 adult females, which seems a figure rather lower than expected.

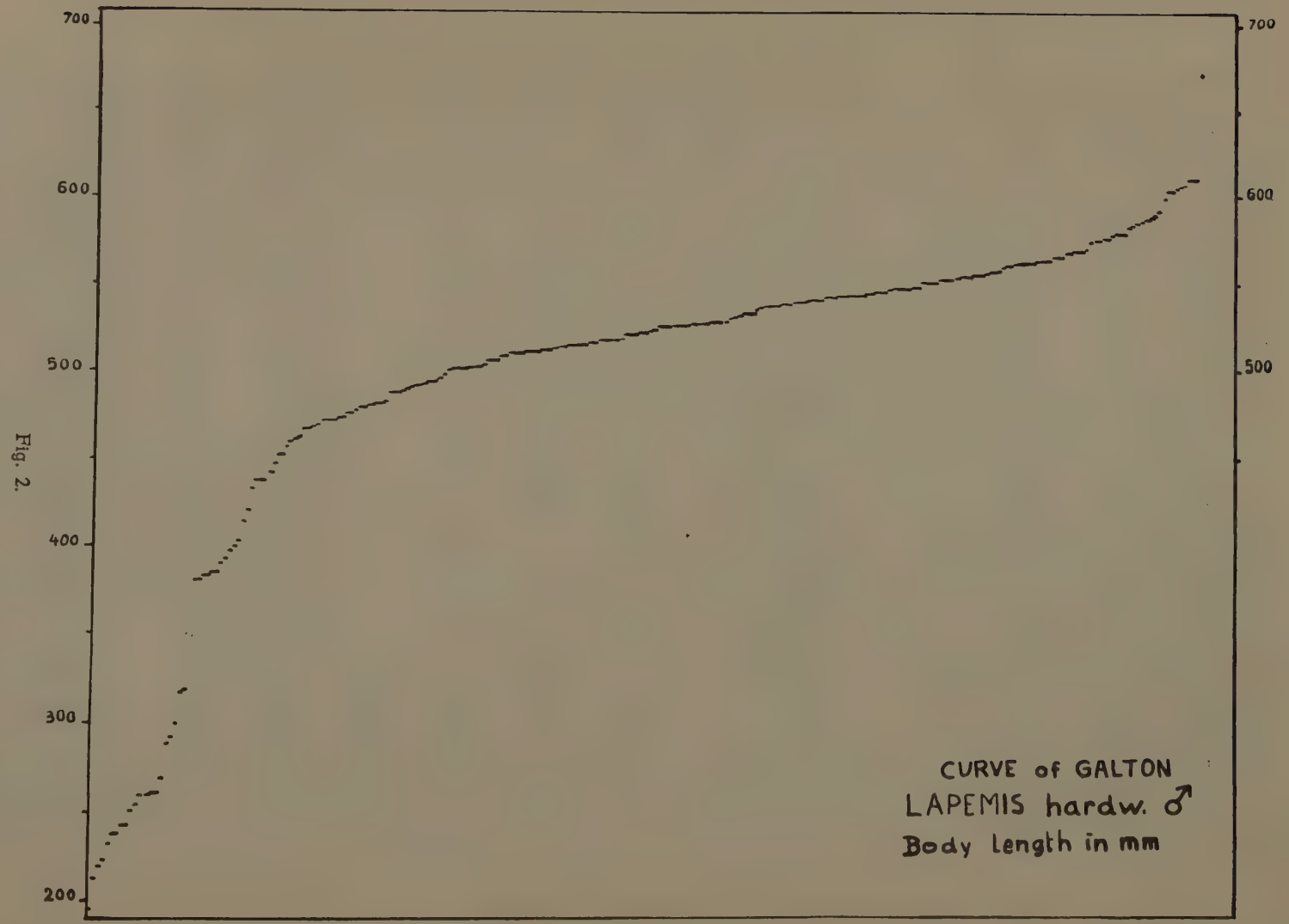
Birth, Maturity.

In the series of WALL, young are born between May and August, foetuses being found in June and July, measuring from 215 to 355 mm. In the Surabaia series birth occurs later, between October and December, at the end of the dry season, November being generally considered as the hottest month of the whole year.

In a somewhat large series, maturity is indicated by the length of the shortest pregnant female, which in our series is an animal with a body length of 449 mm. The first bend in the curve of GALTON occurs exactly at the same place and in the case of Lapemis this is the same spot for males and females. This point may be considered as the treshold between immature and mature animals, between youth and adult age.

Growth.

There are no data of actually measured growth. WALL mentions an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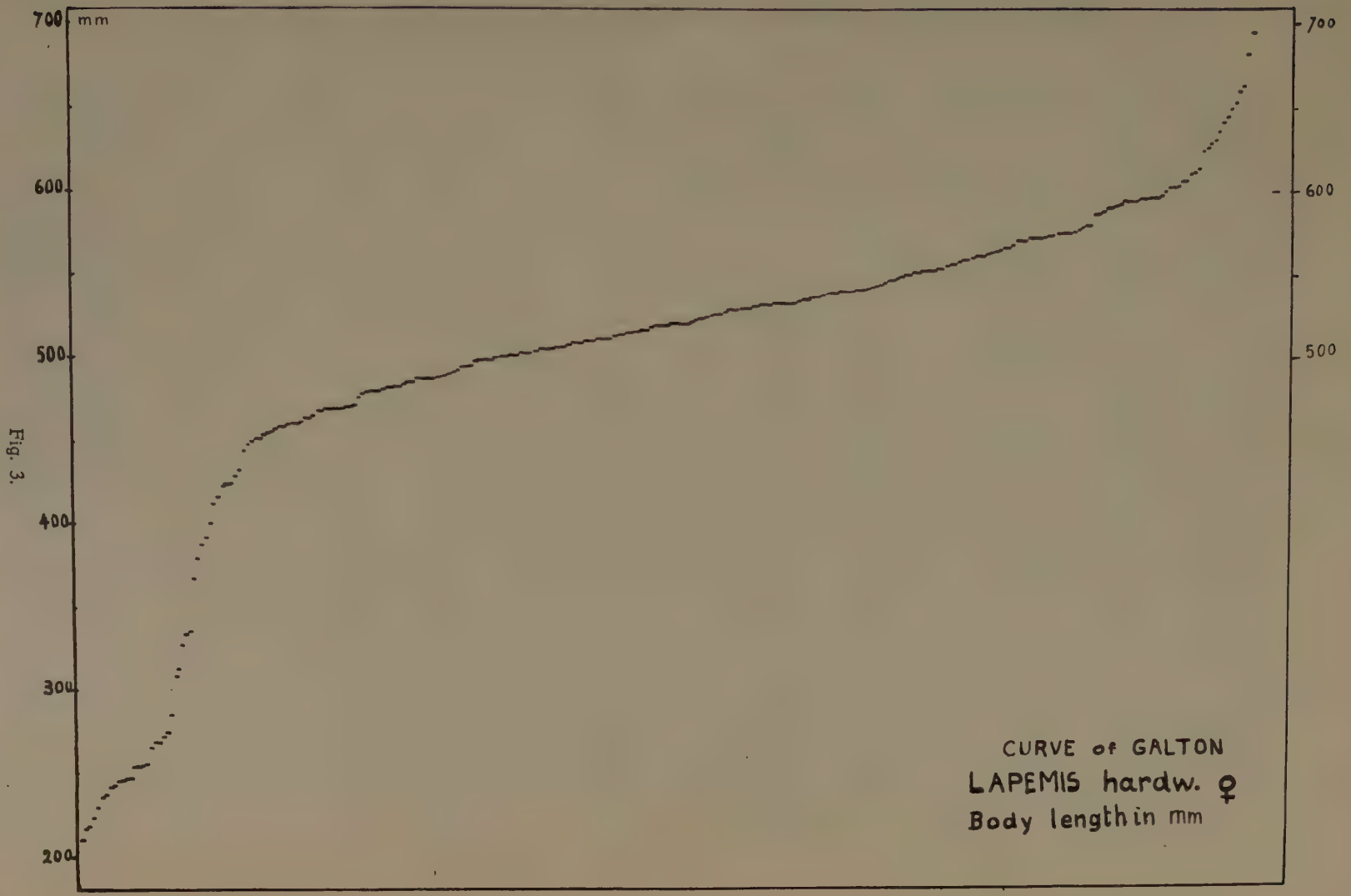
CURVE of GALTON
LAPEMIS hardw. ♂
Body length in mm

Fig. 2.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400
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200
700
600
500
CURVE of GALTON
LAPEMIS hardw. ♀
Body length in mm

Fig. 3.

increase of six to eight inches in the first year. The figures in the Surabaia series may suggest a somewhat slower development. The bodylength of the smallest newborn male is 190 mm and the female 200 mm, the body length of the mature animals is 450 mm. Between those points however, the specimina are not evenly distributed but seem to belong to two different groups showing the same range for both sexes. Males first from 190 to 320 and second from 370 to 400, females first from 200 to 340 and second from 370 to 400 mm.

The first group is distributed over the months October to March, the second from May to November, overlapping each other only during October—November. In these overlapping periods the smallest animals are clearly newborn so it may be concluded that it must have taken the others a year to reach the size they show.

We will see later that in adult Lapemis there is no difference in size in both sexes, so it seems justifiable in the early stages of life to neglect the sex difference and to calculate the average body length in each juvenile group for males and females together. These averages will then be for the first juvenile group including the newborn animals 259 mm and for the second juvenile group 382 mm. In our hypothesis the difference of 123 mm is the growth in the first year. The further growth from the average bodylength of the one year old to the mature size, from 382 mm to 450 mm, is 70 mm and it can reasonably be estimated that these are gained in the second year of life.

In this line of thought, the growth rate as given by SIMPSON & ROE in the formula

$$G = 2{,}3026 \frac{\log Yt - \log Yo}{t}$$

will be in weekly % for the first year of life 0,065 and for the second year 0,053. Or if we prefer to consider growth as uniform (in two years) from birth to maturity, 0,038.

Next to using the average sizes of different groups as an indicator of growth, we can check up the records of single female individuals. In this latter method we have not only the body length to go by, but also the aspect of the ovaries, which presents a very definite picture of different stages of activity correlated with seasons and with age.

In our series a newborn female, caught in December had only thin colourless peritoneal folds without any morphological differentiation, but identifiable as ovaries, her body length was 250 mm. Another young female with a bodylength of 328 mm was caught in February, it was the smallest one to show yellow pin-points in the ovaries as the first sign of eggs. In a third animal of 389 mm bodylength, caught in October, the little eggs were clearly visible in a regular straight row, as neat little spheres each with a diameter of nearly 3 mm. Clearly it was still an immature animal, but with the eggs on the verge of ripening and it took at least

from December till October to reach this stage, together with a growth from 250 to 390 mm in bodylength. Finally the smallest pregnant female was caught in August with a bodylength of 449 mm. In this small series the differences in size between the animals from December and October was 140 mm and from October to August 60 mm. This is 200 mm in round 20 months or 87 weeks and shows a growth rate of 0,047 % weekly, which is on good accordance with the figures found by the other method.

In the Surabaia series the number of firstyear animals is 63, and of the two year olds 14. If this is to be considered as the constant pattern of the population, it would suggest for the juvenile animals in the course of the first year a death rate of about two thirds.

Blood, Pathology.

In 1937 RADSMA and in 1939 STREEF analysed samples of blood from Lapemis, from the latter series only two records could be saved from the war plunder. The figures are shown in the following table.

Blood salt in mg % of plasma *Lapemis*

Author	Sex	Na	K	Ca	Total Cl
RADSMA 1937	—	421	35,–	—	—
STREEF 1939	Fem	433	35,–	—	469
STREEF 1939	Fem	410	35,5	—	483

There are only a few data on pathology. In the Surabaia series there were two abnormalities with the aspect of a tumor, one in a female on the back and one in a male on the eye, also a few ulcerating wounds of the skin. Infection with parasitic worms is very frequent, mostly under the skin and in the wall of the stomach. Of the organs, the kidneys, the testicles and the uterus showed macroscopical abnormalities. In two females one kidney was missing, once left and once right, in two other females the right kidney was abnormally thin. In three females the uterus was abnormal on the distal end, twice on the left side, once not marked. In males there were no kidney abnormalities, but in 10 cases the right testicle was abnormally thin, the bodylength of these animals ranging from 383 to 673 mm.

Length and Weight.

In the next table the maximum lengths given in litterature and my own data are reproduced. For the data of BOURRET and my own I calculated the mean value. The figures of WALL, SMITH and BOURRET seem to belong to one group, different from the others.

If we ask for the probable maximum length, the answer for the Surabaia series is that not more than three animals in a thousand will exceed 647 mm for males in bodylength or 682 for females. In fact we found one male in 220 longer than 647 mm and two females in 243 longer than 682 mm.

We have already mentioned that there is no easily recognisable sexual

Length in mm *Lapemis*.

Author		Sex	Total	Body	Tail
		Maximum			
STEJNEGER	1907	—	750	670	80
DE ROOY	1916	—	750	670	80
WALL	1921	—	846	—	—
TAYLOR	1922	—	762	689	73
SMITH	1926	—	860	785	75
BOURRET	1935	—	860	770	80
BERGMAN		M	753	673	80
		F	777	697	80
		Averages			
BOURRET		—	605	542	63
BERGMAN	adults	M	599	533	66
	„	F	607	535	64

dimorphism in Lapemis, nor is there a difference in bodylength or in weight. The average bodylength is in males 533, and 535 mm in females, the weight in adult males 161 grams and 163 grams in female adults.

In the length of the tail however there seems to be a definite difference, the male adults have longer tails than the adult females 66,2 mm against 63,4 with $D/\sigma D = 4{,}6$ which is significant.

The coefficient of correlation r for body length and tail length is in adult males 0,676 and in adult females 0,757; the same coefficient for bodylength and weight is in adult males 0,646 and in adult females 0,756.

Sexual Dimorphism *Lapemis* adult.

	Sex	N	R	M	σ	D/σD
Weight	M	137	68—279	161 ± 3,2	45 ± 2,7	2/5,7 = 0,35
	F	122	78—340	163 ± 4,7	52,5 ± 3,4	
Body	M	220	450—673	533 ± 2,6	37,9 ± 1,8	1,9/4,7 = 0,46
length	F	245	450—697	534,9 ± 3,2	47,5 ± 2,2	
Tail	M	220	54—80	66,2 ± 0,4	5,2 ± 0,3	2,8/0,6 = 4,6
length	F	245	52—80	63,4 ± 0,4	5,6 ± 0,3	

Length of internal organs.

The lengths of the heart, liver, gallbladder, kidneys, are in Lapemis of both sexes of the same order. The testicles measure less than half the length of the ovaries. The length of spleen and pancreas seem to be greater in males, the D/σD being for the spleen 4 and for the pancreas 4,5. It is however not at all sure that this statistically significant difference has any special biological importance, precisely these organs are during pregnancy markedly pushed cranially; often then the pancreas can be seen turned to the left, so that its long axis is no longer in line with the long axis of the body. When the most cranial and caudal points of the pancreas are measured then, they do not give the longest diameter of the oval as they should do and the organ seems shorter than it really is.

Topography.

The analysis of the data given in the table shows a difference between the topography of the organs in adult males and females. The heart, liver,

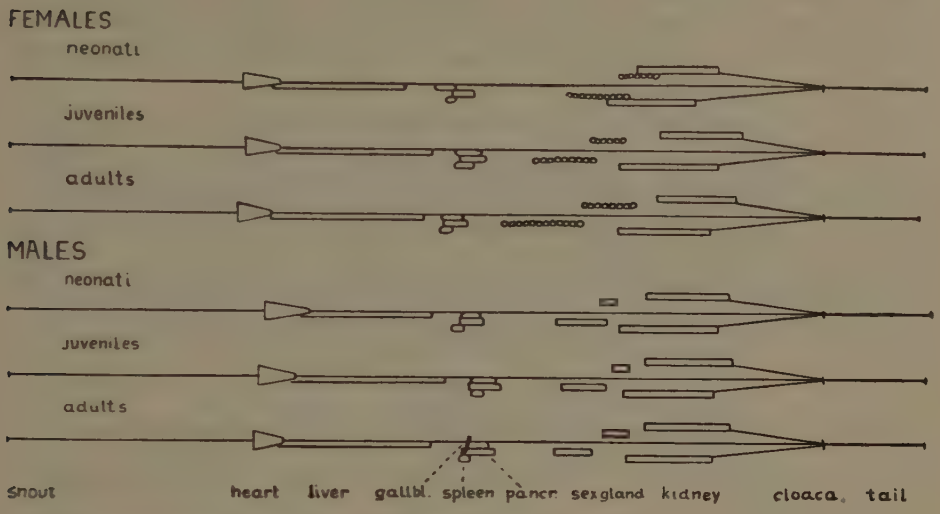

Fig. 4.

gallbladder, spleen and pancreas, in short all the organs of the cranial half of the body are more cranially placed in females. The position of the kidneys on the contrary is in both sexes the same and so it is with the caudal end of the sex glands on the right and left side to. Seeing the difference in length of the sex glands in males and females it is evident that the cranial end of the ovaries must extend much further forwards than that of the testicles. The differences and their sigma are listed in the following table. No measures of the lung have been listed, as in these animals, the lung stretches over the whole length of the body.

Sexual Dimorphism in topography D/♂D *Lapemis* adult

	Top	End
Heart	9,2/1,2 = 7,7	8,6/1,3 = 6,6
Liver	8,0/1,4 = 5,7	6,3/2,2 = 2,9
Gall bladder	17,4/2,3 = 7,5	16,4/2,5 = 6,6
Spleen	14,9/2,4 = 6,0	15,6/2,4 = 6,5
Pancreas	15,0/2,4 = 6,2	16,8/2,6 = 6,5
Sex gland R	25,6/2,8 = 9,2	
id L	14,6/3,1 = 4,8	
Sex gland R		4,3/3,1 = 1,4
id L		6,8/3,5 = 1,9
Kidney R	5,2/3,2 = 1,6	2,7/1,2 = 2,2
Kidney L	6,4/3,5 = 1,8	2,8/1,2 = 2,3

Intervals between the organs.

In Lapemis heart and liver are so near to one another that there is hardly an interval at all and there is no noticeable difference between both sexes. Gallbladder, spleen and pancreas are also practically fused into one block. The two remaining intervals in the cranial half of the body, the distance from snout to heart and from liver to gallbladder are shortened in female animals.

For the caudal half of the body one has to distinguish between right and left side. On the left side the intervals do not show any sexual difference, on the right side on the contrary there is a difference between males and females. The most important seems to be the distance between pancreas and kidney which is longer in female animals. This is quite understandable as this is the space where in female animals one of the most important functions, the development of the young, has to be carried out.

Sexual Dimorphism in organ intervals. *Lapemis adult*

	Sex	M		D/σD
Snout-heart	M	158,0	0,8	
	F	148,8	0,9.	10,8/1,21 = 8,9
Liver-gallbl.	M	22,0	0,7	
	F	10,9	0,7	11,1/1,— = 11,—
Pancreas-sex gl	M	39,1	0,7	
	F	20,3	0,7	18,8/1,— = 18,—

Variability.

The variability of the bodyconstituents thus far analysed, shows very different values. The length of body and tail, the topographical data of the pattern of the internal organs, the sum of all intervals on both sides of the body are not very variable, the coefficient ranging from 7 to 12 for both sexes.

The lengths of different organs however show two distinct types of variability. V from 11 to 15 for heart, liver and kidney, but from 18 to 26 for gallbladder, pancreas and spleen. The sexglands are even more variable, for males right and left V = 24 and 29, for females 26 and 35. If these glands of both sides are taken together as a unity, as they functionally undoubtedly are, the variability diminishes, but then the testicles belong to the first group of organs with V = 12, and the ovaries with V = 24 rather to the second.

The intervals or free spaces show a different variability for those on the ends of the body and those in between. The cranial interval, snout-heart, and the caudal ones, kidney-cloaca, right and left, have a coefficient from 8 to 14. For all other intervals the variability is uncommonly high, with the single exception of the interval between pancreas and left sexgland.

This seems to point first to a very strong tendency to keep the topo-

graphical pattern constant, secondly to a marked tendency also to keep the length of the organs constant although the organs at the middle of the body seem to have more freedom in this respect than the others, and thirdly, to the fact the arrangement of the free spaces or intervals between the organs seems in every single case very uncertain, although as a whole they show very little freedom.

Summary.

An analysis of various data on the seasnake Lapemis hardwickei has been given. About 550 specimina were collected from 1936 to 1942 at Surabaia, East Java. The growth of the young individuals, the point where maturity is reached, the fertility and the date of birth, the pathology and the chemical analysis of the blood, the sexual dimorphism as shown by the data have been discussed and a detailed account of the topographical anatomy of this snake has been given.

The tables are printed on p. 894—898 of this number.

Batavia, January 1949.

Anatomy. — *The Anatomy of Lapemis Hardwickei gray.* II. By R. A. M. BERGMAN. (Department of Anatomy, Batavia.) (Communicated by Prof. J. BOEKE.) [1])

(Communicated at the meeting of June 25, 1949.)

Lapemis hardw. Body length of young

Month	M	F	M	F	Month	M	F	M	F
October	215	208		368	January	256	247	347	393
	240	246		389		298	255		
	253	255					268		
November	210	234	398		February	314	275		
	248	217					314		
December	195	215	396				328		
	223	223			March	286	286		380
	230	229				290	309		
	235	236				316	336		
	240	242			April				
	255	243			May			383	
	256	246			June			380	
	257	247			July			378	
	257	256						392	
	268	258			August			377	
		266			September	235		380	
		268						383	
		272			N	22	27	10	4
					M	255	262	381	382

[1]) The tables and lists in the following pages refer to the article "The Anatomy of Lapemis Hardwickei gray" where these data have been discussed.

Topography		*Lapemis* hardwickei		*Adult males*	
top of	N	R	M	σ	V
Heart	190	126/188	158,0 ± 0,8	11,3 ± 0,6	7,2 ± 0,4
Liver	190	142/212	176,4 ± 0,9	12,5 ± 0,6	7,1 ± 0,4
Gallbladder	190	248/359	299,0 ± 1,5	20,8 ± 1,1	7,0 ± 0,4
Pancreas	190	250/350	297,6 ± 1,5	21,0 ± 1,1	7,0 ± 0,4
Spleen	190	252/354	295,6 ± 1,6	21,4 ± 1,1	7,2 ± 0,4
Testis R.	190	295/420	356,7 ± 1,9	25,6 ± 1,3	7,4 ± 0,4
Testis L.	191	330/471	390,4 ± 1,9	26,8 ± 1,4	6,9 ± 0,4
Kidney R.	188	340/492	404,2 ± 2,0	27,1 ± 1,4	6,7 ± 0,3
Kidney L.	189	354/516	417,3 ± 2,2	30,7 ± 1,6	7,4 ± 0,4
end of					
Heart	190	147/214	179,5 ± 0,9	12,0 ± 0,6	6,7 ± 0,3
Liver	190	228/326	276,9 ± 1,5	20,2 ± 1,0	7,3 ± 0,4
Gallbladder	190	263/374	314,0 ± 1,6	22,4 ± 1,2	7,1 ± 0,4
Pancreas	190	269/376	317,3 ± 1,7	23,5 ± 1,2	7,4 ± 0,4
Spleen	190	257/361	305,4 ± 1,5	21,0 ± 1,1	7,1 ± 0,4
Testis R.	190	323/461	381,0 ± 2,0	26,8 ± 1,4	7,1 ± 0,4
Testis L.	191	342/504	405,1 ± 2,1	29,2 ± 1,5	7,2 ± 0,4
Kidney R.	188	383/578	462,3 ± 2,5	34,0 ± 1,8	7,4 ± 0,4
Kidney L.	189	397/595	475,1 ± 2,5	34,2 ± 1,8	7,2 ± 0,4
Cloaca	220	450/673	533,0 ± 2,6	37,9 ± 1,8	7,1 ± 0,4

Topography		*Lapemis* hardwickei		*Adult females*	
top of	N	R	M	σ	V
Heart	212	122/202	148,8 ± 0,9	13,0 ± 0,6	8,6 ± 0,4
Liver	212	140/223	168,4 ± 1,0	14,3 ± 0,7	8,5 ± 0,4
Gallbladder	211	224/381	281,5 ± 1,8	25,7 ± 1,3	9,5 ± 0,5
Pancreas	212	227/376	282,6 ± 1,8	26,4 ± 1,3	9,4 ± 0,5
Spleen	212	222/385	281,3 ± 1,9	27,4 ± 1,3	9,7 ± 0,5
Ovary R.	212	252/408	321,1 ± 2,1	30,6 ± 1,5	9,4 ± 0,5
Ovary L.	212	304/508	375,8 ± 2,4	35,0 ± 1,7	9,3 ± 0,5
Kidney R.	211	326/534	398,9 ± 2,5	36,4 ± 1,8	9,1 ± 0,4
Kidney L.	212	347/565	423,7 ± 2,7	39,3 ± 1,9	9,7 ± 0,5
end of					
Heart	212	140/228	171,0 ± 1,0	14,1 ± 0,7	8,3 ± 0,4
Liver	212	219/338	270,6 ± 1,7	24,0 ± 1,2	8,8 ± 0,4
Gallbladder	211	238/400	297,6 ± 1,9	27,2 ± 1,3	9,1 ± 0,4
Pancreas	212	236/396	300,5 ± 1,9	27,9 ± 1,4	9,3 ± 0,5
Spleen	212	229/392	289,8 ± 1,9	27,2 ± 1,3	9,4 ± 0,5
Ovary R.	212	307/515	377,7 ± 2,5	35,8 ± 1,7	9,5 ± 0,5
Ovary L.	212	330/549	411,9 ± 2,8	40,6 ± 2,0	9,8 ± 0,5
Kidney R.	211	379/604	459,6 ± 3,0	43,0 ± 2,1	9,3 ± 0,5
Kidney L.	212	398/630	478,0 ± 2,9	43,2 ± 2,1	9,6 ± 0,5
Cloaca	245	450/697	534,9 ± 3,2	47,5 ± 2,1	8,9 ± 0,4

Length of	N	*Lapemis* hardwickei R	M	*Adult females* σ	V
Body	245	450/697	534,9 ± 3,2	47,5 ± 2,1	8,9 ± 0,4
Tail	243	52/80	63,6 ± 0,4	5,8 ± 0,3	9,1 ± 0,4
Heart	212	16/30	22,0 ± 0,2	2,9 ± 0,1	13,2 ± 0,6
Liver	212	75/152	102,6 ± 0,9	13,4 ± 0,6	13,0 ± 0,6
Gallbladder	211	8/25	15,7 ± 0,3	3,4 ± 0,2	21,7 ± 1,0
Pancreas	212	8/34	18,0 ± 0,3	3,9 ± 0,2	21,5 ± 1,0
Spleen	212	2/15	8,5 ± 0,2	2,2 ± 0,1	25,8 ± 1,2
Ovary R	212	18/109	57,1 ± 1,0	14,8 ± 0,7	25,8 ± 1,2
Ovary L.	211	4/81	36,3 ± 0,9	12,7 ± 0,6	35,3 ± 1,6
Both ovaries	210	22/165	93,1 ± 1,6	22,7 ± 1,0	24,4 ± 1,1
Kidney R.	212	30/99	64,3 ± 0,8	11,0 ± 0,5	17,1 ± 0,8
Kidney L.	212	32/88	55,1 ± 0,6	8,0 ± 0,4	14,5 ± 0,7
Both kidneys	212	68/182	120,2 ± 1,2	17,3 ± 0,8	14,3 ± 0,6
Weight	122	78/340	163,0 ± 4,7	52,5 ± 3,4	34,3 ± 2,2

Length of	N	*Lapemis* hardwickei R	M	*Adult males* σ	V
Body	220	450/673	533,0 ± 2,6	37,9 ± 1,8	7,1 ± 0,4
Tail	220	54/80	66,2 ± 0,4	5,2 ± 0,3	7,9 ± 0,4
Heart	190	16/32	21,7 ± 0,2	3,0 ± 0,2	13,9 ± 0,7
Liver	190	65/133	100,3 ± 0,8	11,5 ± 0,6	11,3 ± 0,6
Gallbladder	190	9/25	15,0 ± 0,2	2,7 ± 0,1	18,0 ± 0,9
Pancreas	190	11/32	19,9 ± 0,3	3,9 ± 0,2	19,6 ± 1,0
Spleen	190	4/19	9,4 ± 0,2	2,3 ± 0,1	25,0 ± 1,3
Testis R.	190	13/39	24,3 ± 0,4	5,8 ± 0,3	23,9 ± 1,2
Testis L.	191	6/24	14,5 ± 0,3	4,2 ± 0,2	28,7 ± 1,5
Both testes	190	21/74	44,0 ± 0,4	5,5 ± 0,3	12,4 ± 0,6
Kidney R.	188	37/91	61,2 ± 0,6	8,5 ± 0,4	13,9 ± 0,7
Kidney L.	189	43/79	57,4 ± 0,6	7,5 ± 0,4	13,1 ± 0,7
Both kidneys	188	86/165	119,0 ± 1,1	15,12 ± 0,8	12,7 ± 0,6
Weight	137	68/279	161,6 ± 3,8	45,0 ± 2,7	28,0 ± 1,7

Intervals from/to	N	*Lapemis* hardwickei R	M	*Adult females* σ	V
Snout/heart	212	122/202	148,8 ± 0,9	13,0 ± 0,6	8,6 ± 0,4
Heart/liver	212	—8/+6	—2,8 ± 0,1	1,9 ± 0,1	71,0 ± 3,4
Liver/gallbl.	212	—15/+43	10,9 ± 0,7	9,6 ± 0,5	87,0 ± 4,2
Total A	212	116/240	157,6 ± 1,2	18,0 ± 0,9	11,6 ± 0,6
Pancreas/ov. R	211	—3/+43	20,3 ± 0,7	10,3 ± 0,5	50,7 ± 2,5
ov. R./kidney R.	210	—10/+69	21,9 ± 1,0	14,3 ± 0,7	68,0 ± 3,0
kidn. R./cloaca	212	45/99	71,6 ± 0,7	10,4 ± 0,5	14,5 ± 0,7
Total B	210	70/163	113,1 ± 1,3	18,9 ± 0,9	16,7 ± 0,8
Pancreas/ov. L.	211	22/118	75,0 ± 1,1	15,9 ± 0,8	21,3 ± 1,2
ov. L./kidney L.	211	—29/+58	11,4 ± 0,8	12,2 ± 0,6	107,0 ± 5,2
Kidn. L./cloaca	212	39/85	56,6 ± 0,5	7,6 ± 0,4	13,5 ± 0,6
Total C	211	90/207	142,5 ± 1,5	21,2 ± 1,0	14,9 ± 0,7
Total A + B	210	214/391	253,8 ± 2,0	28,8 ± 1,4	11,3 ± 0,6
Total A + C	211	225/435	299,5 ± 2,2	32,4 ± 1,6	11,0 ± 0,5

Intervals from/to	N	*Lapemis* hardwickei R	M	*Adult males* σ	V
Snout/heart	190	126/188	158,0 ± 0,8	11,3 ± 0,6	7,2 ± 0,4
Heart/liver	191	—10/+4	—2,8 ± 0,2	2,1 ± 0,1	74,0 ± 3,8
Liver/gallbl.	191	+2/+45	22,0 ± 0,7	9,0 ± 0,5	41,0 ± 2,1
Total A	191	135/221	177,1 ± 1,2	15,9 ± 0,8	9,0 ± 0,5
Pancreas/te. R.	190	11/63	39,1 ± 0,7	9,3 ± 0,5	23,8 ± 1,2
Test. R./kidn. R.	188	—2/+50	20,1 ± 0,6	8,5 ± 0,4	42,3 ± 2,2
Kidn. R./cloaca	188	43/95	69,1 ± 0,6	8,8 ± 0,5	12,7 ± 0,7
Total B	188	81/179	128,8 ± 1,2	15,7 ± 0,8	12,2 ± 0,6
Pancreas/te. L.	191	36/104	73,4 ± 0,8	11,0 ± 0,6	14,8 ± 0,8
Test. L./kidn. L.	190	—3/+35	11,7 ± 0,4	5,9 ± 0,3	53,0 ± 2,7
Kidn. L./cloaca	190	38/79	57,4 ± 0,5	6,8 ± 0,4	11,9 ± 0,6
Total C	190	91/194	142,4 ± 1,1	15,5 ± 0,8	10,9 ± 0,6
Total A + B	188	232/379	305,3 ± 2,0	26,8 ± 1,4	8,8 ± 0,5
Total A + C	191	233/396	319,4 ± 2,0	27,6 ± 1,4	8,6 ± 0,4

Topography in % of bodylength	*Females*			*Males*		*Lapemis* hardwickei
	Neon.	Juven.	Adult	Neon.	Juven.	Adult
Body length	100,0	100,0	100,0	100,0	100,0	100,0
Tail length	12,8	12,3	11,9	13,4	12,6	12,4
Heart top	28,4	28,7	27,8	31,4	30,6	29,6
end	33,0	33,0	32,0	36,7	35,0	33,6
length	4,5	4,3	4,1	4,2	4,2	4,0
Liver top	32,1	32,5	31,5	35,6	34,2	33,1
end	48,5	51,7	50,6	51,7	53,3	51,8
length	16,4	19,2	19,1	15,9	19,1	18,9
Gallbl. top	51,8	54,4	52,6	55,4	56,6	56,0
end	54,3	57,5	55,5	57,8	59,5	58,9
length	2,6	3,1	3,0	2,3	2,9	2,8
Pancreas top	53,9	54,9	52,8	55,0	56,5	55,8
end	56,9	58,2	56,0	58,0	60,3	59,3
length	3,0	3,4	3,2	2,8	3,6	3,7
Spleen top	53,1	54,3	52,5	54,1	56,5	55,4
end	54,2	56,8	54,0	55,3	58,0	56,6
length	1,1	1,5	1,5	1,1	1,5	1,8
R. sexgland top	68,0	64,0	60,0	67,0	67,6	66,8
end	76,0	71,8	70,5	71,2	71,2	71,3
length	7,9	7,7	10,6	4,1	3,7	4,6
L. sexgland top	74,5	70,8	70,2	72,3	74,0	73,0
end	79,3	75,3	77,0	74,6	76,0	76,0
length	4,9	4,6	6,7	2,3	1,9	2,8
R. kidney top	72,7	74,3	74,5	74,7	75,5	75,7
end	84,0	87,0	86,0	86,6	87,0	86,6
length	11,2	12,8	11,4	11,8	11,4	10,9
L. kidney top	76,7	79,3	79,0	78,0	78,0	78,0
end	86,9	90,0	89,3	88,7	89,0	89,0
length	10,1	10,6	10,2	10,4	11,0	10,8
Bodylength in mm	267,0	416,0	535,0	254,0	404,0	533,0

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Physics. — *Solar flares and the origin of Cosmic Radiation.* I. By J. CLAY, H. F. JONGEN and A. J. DIJKER. (Natuurkundig Laboratorium University of Amsterdam.)

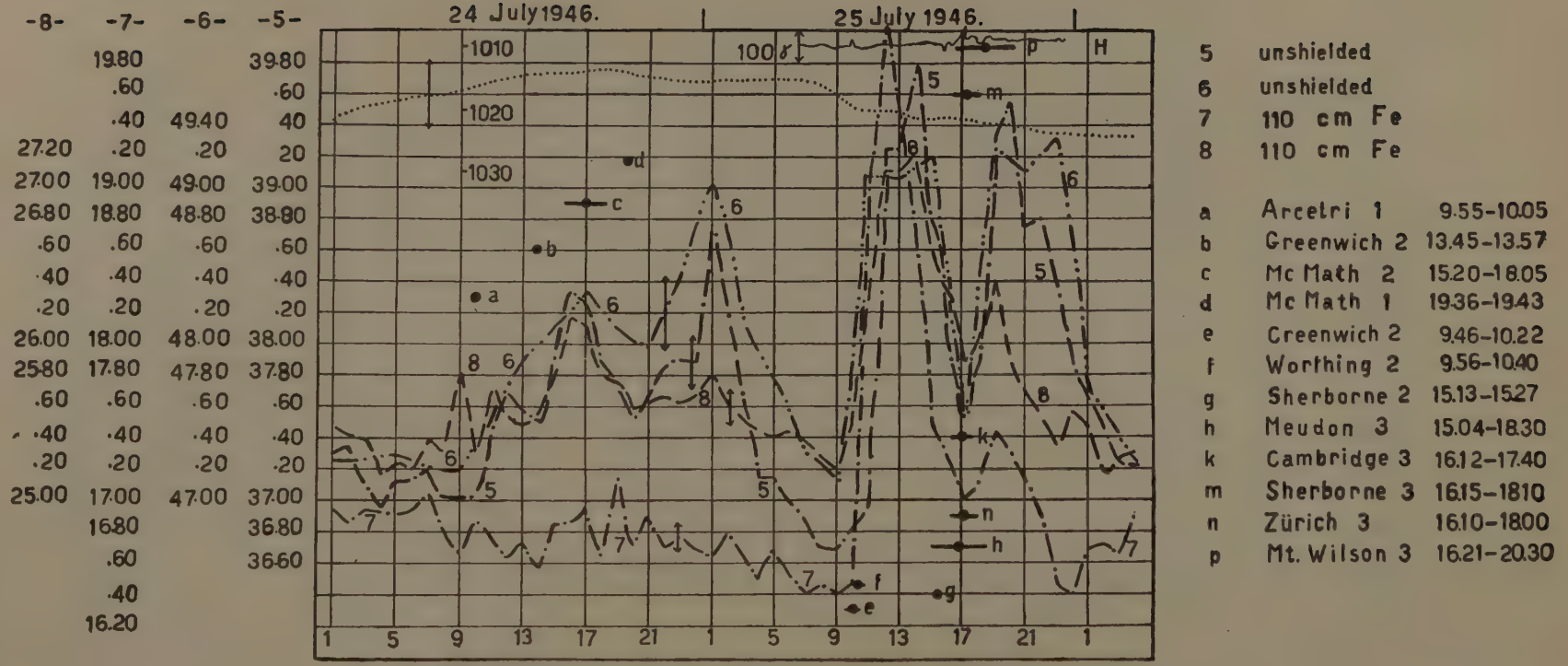
(Communicated at the meeting of September 24, 1949.)

On July 25, 1946 a considerable increase of cosmic radiation was measured at different places of the earth [1, 2]). This was also the case in Amsterdam where for the last 15 years the radiation has been continuously recorded with four ionisation vessels of 40 L each, filled with argon of pressure of 40, 40, 97 and 58 atmosphere resp. The hour-values of the recordings have an accuracy of 0.2 %. One vessel is unshielded (6), one has generally a shield of 12 cm (on the 25th the shield was absent), the remaining two have a shield of 110 cm Fe. In the graph 1 we see the 3 hour-values of these ionisation-vessels. The barometer-curve is given below in millibars, the variation with barometric pressure is from 1.7 % per cm Hg (under 110 cm Fe) to 3 % per cm Hg (open). In the middle of graph 1 the horizontal component of the earth's magnetic field is given in γ units and the variation of 100 γ is indicated by the arrow.

VALLARTA has expounded that the increase of the cosmic radiation might be caused by a magnetic dipole-field of two sunspots, which would compensate the sun's magnetic field, so that at that moment electrically charged particles would be able to escape from the sun, which they could not do as long as these sun spots did not occur. However, we have some doubt as to the validity of this explanation.

In the graph 1 we see an increase of the intensity in the four apparatus between 10 and 15 h. G.M.T. on the 25th of July, but then on July 26 a heavy magnetic storm sets in and in all four vessels a decrease of intensity is recorded, as is usual in the event of magnetic storms. We may mention that on July 25 at 10 a little peak is found in the horizontal component of the geomagnetic field. We have noticed that this is a characteristic feature in practically all the cases where solar flares are accompanied by an increase of cosmic radiation.

We next took the hour-values in stead of the 3-hour values and proceeded to a closer examination of the increases. We found that in the more precise data the variations showed much more plainly and were found to occur as in graph 2. Moreover we detected maxima on the 24th of July, a day of uncommonly heavy activity on the sun, apparently from a great number of solar flares, observed at different stations. The flares are indicated in graph 2 by *a*—*p*. The name of the observatory [3]), the duration and the class of the flares are given in the right-hand part of the graph. For classification of intensity the numbers 1, 2 and 3 are used. Especially the flares *h* till *p* inclusive were of high intensity.



5	unshielded	
6	unshielded	
7	110 cm Fe	
8	110 cm Fe	
a	Arcetri 1	9.55–10.05
b	Greenwich 2	13.45–13.57
c	Mc Math 2	15.20–18.05
d	Mc Math 1	19.36–19.43
e	Greenwich 2	9.46–10.22
f	Worthing 2	9.56–10.40
g	Sherborne 2	15.13–15.27
h	Meudon 3	15.04–18.30
k	Cambridge 3	16.12–17.40
m	Sherborne 3	16.15–18.10
n	Zürich 3	16.10–18.00
p	Mt. Wilson 3	16.21–20.30

Graph. 2. One hour recordings of Cosmic Radiation in 4 ionisation chambers of 40 L, on July 24 and 25, 1946 with solar flares observed at different stations. Barometer.

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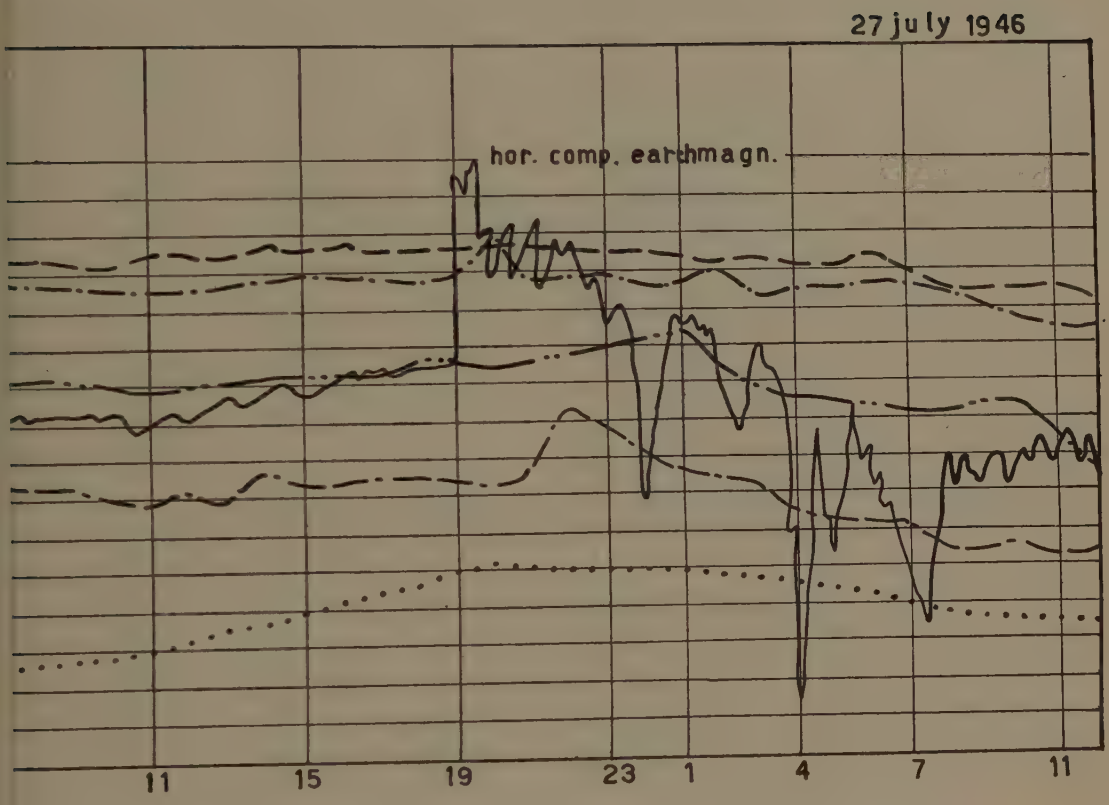
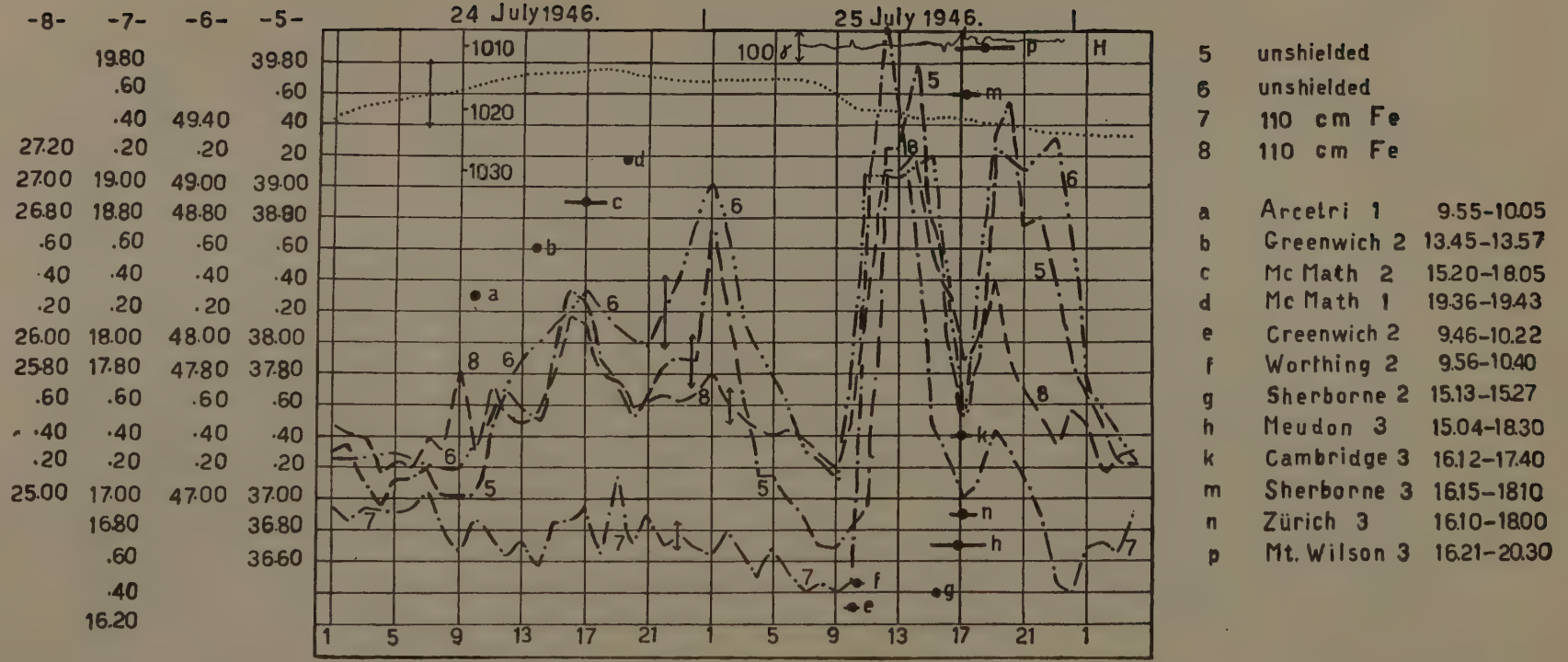


40 L argon at high pressure. Increase on July 25, 1946, during solar flares, decrease on
Hc storm. Barometer.



5	unshielded	
6	unshielded	
7	110 cm Fe	
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a	Arcetri 1	9.55–10.05
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h	Meudon 3	15.04–18.30
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m	Sherborne 3	16.15–18.10
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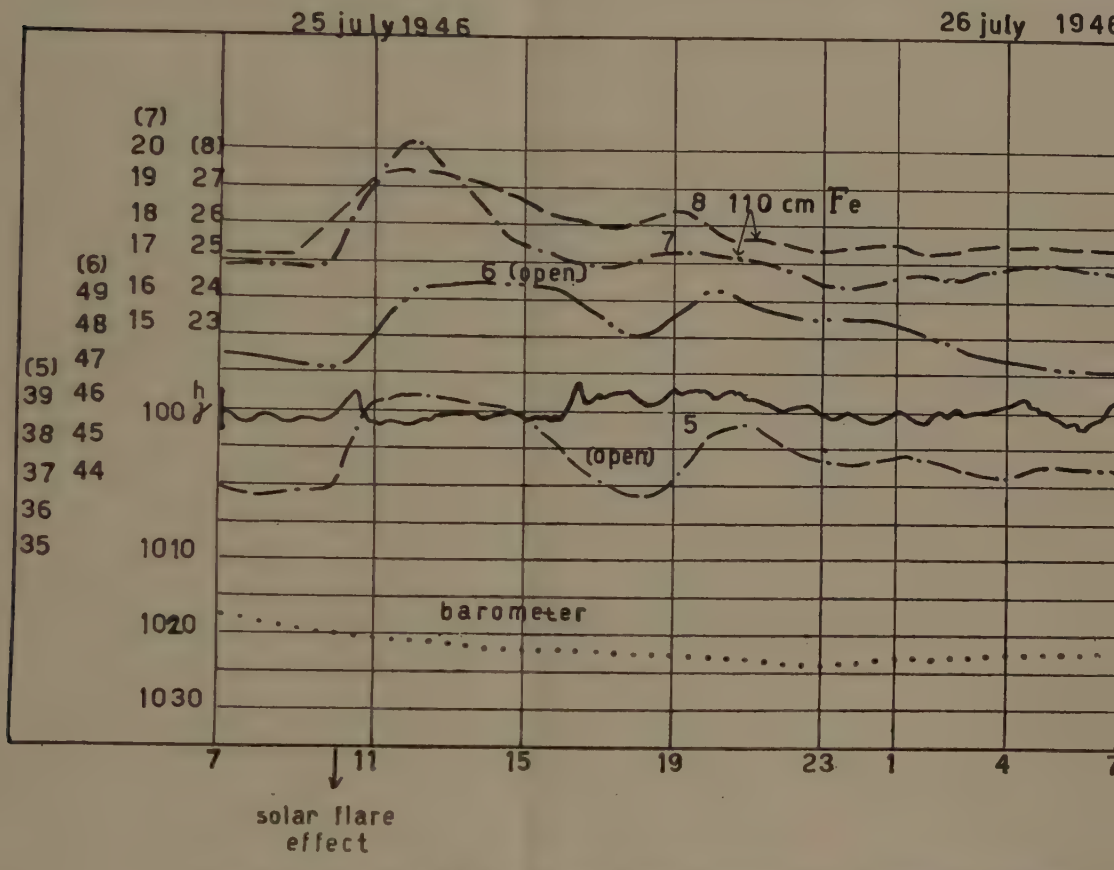
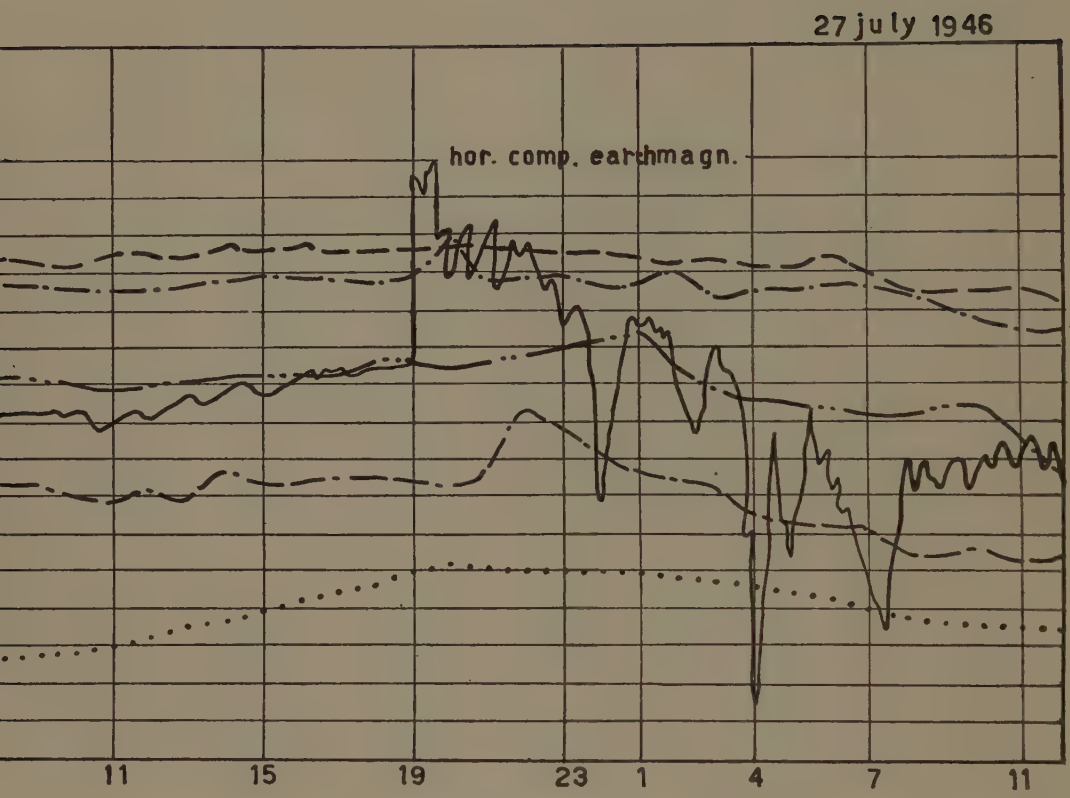
p	Mt. Wilson 3	16.21–20.30

Graph. 2. One hour recordings of Cosmic Radiation in 4 ionisation chambers of 40 L, on July 24 and 25, 1946 with solar flares observed at different stations. Barometer.



Graph. 1. Recording of 3 hours mean Cosmic Radiation with 4 ionisation chambers of
July 27 during magn

on.



0 L argon at high pressure. Increase on July 25, 1946, during solar flares, decrease on
storm. Barometer.

The relative values of the ionisation currents are given at left expressed in compensation-voltage.

After scrutinizing this detailed result we looked for more cases, starting with the year 1947, which was in part of great solar activity. In this year alone we found more than 30 cases of considerable increased radiation and still 30 more of smaller value all correlated with solar flares. In most cases the shift in the maximum occurred up to 2 hours after the flare was observed.

The magnetic curve does not show variations worth mentioning in this period. But sometimes a small peak arises in the horizontal component a short moment before the beginning of the increase.

We took 7 particular cases with increases of 3—5 % of the radiation and we noted the moment of the observation of the flare and the station where it was observed. Then we looked for sun spots which might be correlated with the flares, considering their place and history, which are given in the text below [4, 5]).

The magnetic moment of the spot is calculated from its surface and the maximum value of the magnetic field within it (in the way suggested by BROXON [6]). The magnetic moments are given in the graphs 3—8. The earth's magnetic field is given in the graph for June 17 1947. We see that the computed magnetic moments of the sunspots differ by a factor of 10^{-4} till 10^{-5} from the formerly accepted total magnetic moment of the sun, which was assumed to be 10^{34} Gauss cm^3.

It is possible that the magnetic fields of these spots are important or indispensible to the phenomenon treated above, but our knowledge on the subject is very fragmentary as yet.

A short time ago a publication by C. E. R. BRUCE [7]) and one by M. A. ELLISON [8]) treated of the possibly considerable magnetic field of the flares. In the graphs below the positive variation of the radiation is given together with the time of observation of the flares. In most cases the maximum of the radiation arose with a retardation of 0—2 hours.

The magnetic dipole momenta are calculated with the formula $M = 0.177a^3 H$, given by BROXON [6]) in his recent paper on this subject in the Physical Review, 1949.

a is the radius of the sunspot which we take as circular and H is the maximum value of the magnetic field which is measured in the center of the spot

May 21, 1947	spot number 8566	$M = 4{,}2 \times 10^{29}$ G cm^3
June 17, 1947	spot number 8626	$M = 9.4 \times 10^{28}$ G cm^3
July 14, 1947	spot number 8709	$M = 1.5 \times 10^{29}$ G cm^3
July 24, 1947	spot number 8715	$M = 4.3 \times 10^{30}$ G cm^3
Aug. 11, 1947	spot number 8764	$M = 1.1 \times 10^{30}$ G cm^3
Aug. 15, 1947	spot number 8775	$M = 5.5 \times 10^{28}$ G cm^3

The flares on July 25 1946 were at a latitude of +23° North. In this point particles of 10^{10} eV cannot escape from the sun [1]).

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[1]) Northern latitude is positive. Difference in longitude is positive towards the west.

May 21, 1947.

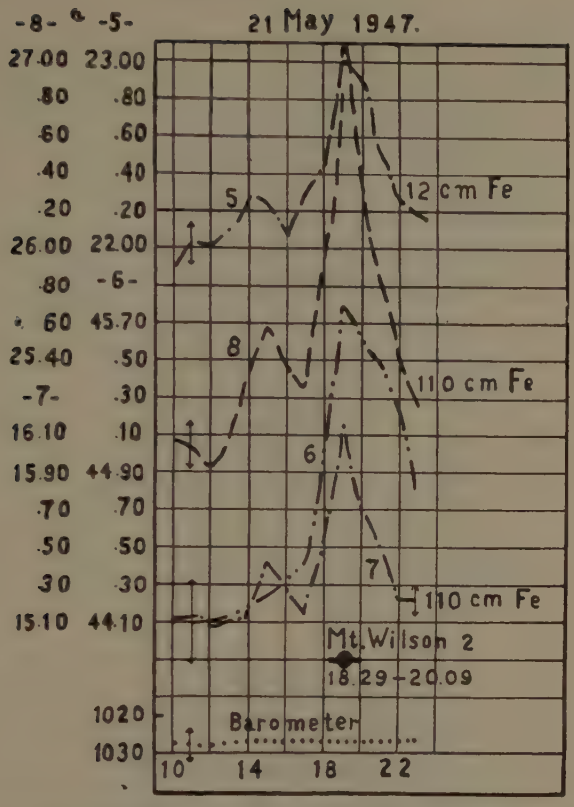
Inset of the increase of intensity in Cosmic Radiation after 18 h. with maximum at about 19 h. Observed solarflare class 2 on Mount Wilson between 18.29 and 20.08 with the maximum 18 h. 44.

The situation of the flare was latitude +20° and the distance till the central meridian —25°.

On this day in the surroundings of the solarflare we mentioned 2 sunspots were observed, 8568 and 8571, with a bipolar magnetic character. The spots are found from 16—29 resp. from. 17—29 May, 1947. Because the spot 8568 is the most likely cause of the effect, we give its history below:

Data	Diff. of latitude to the central meridian	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
May 16, 1947	— 87°	195°	+ 17°	97	1
	— 87°	195°	+ 19°	48	1
May 17, 1947	— 75°	194°	+ 18°	87	6
May 18, 1947	— 65°	190°	+ 19°	194	3
	— 61°	194°	+ 15°	436	5
	— 58°	197°	+ 17°	727	14
May 19, 1947	— 51°	191°	+ 19°	194	14
	— 45°	197°	+ 17°	1067	27
	— 40°	202°	+ 16°	145	9
May 20, 1947	— 19°	190°	+ 20°	170	20
	— 33°	196°	+ 17°	1018	22
	— 27°	202°	+ 17°	48	10
May 21, 1947	— 27°	188°	+ 20°	194	8
	— 20°	195°	+ 19°	630	19
	— 17°	198°	+ 17°	194	13
May 22, 1947	— 11°	191°	+ 18°	97	10
	— 5°	197°	+ 18°	697	32
	0°	202°	+ 17°	121	12
May 23, 1947	0°	189°	+ 18°	194	12
	+ 7°	196°	+ 17°	582	26
	+ 12°	201°	+ 17°	242	14
May 24, 1947	+ 13°	189°	+ 18°	145	15
	+ 20°	196°	+ 17°	533	28
	+ 26°	202°	+ 17°	242	15
May 25, 1947	+ 25°	188°	+ 19°	145	30
	+ 32°	195°	+ 18°	436	50
	+ 38°	201°	+ 18°	170	31
May 26, 1947	+ 48°	197°	+ 18°	485	13
May 27, 1947	+ 56°	192°	+ 21°	48	3
	+ 60°	196°	+ 18°	533	12
	+ 66°	202°	+ 17°	24	5
May 28, 1947	+ 76°	199°	+ 16°	436	4
May 29, 1947	+ 88°	198°	+ 16°	145	2

On May 29, this spot dissappears from the visible region on the west side. The underlined part of the spot was about in the same place as the flare the number of separate spots having diminished from 20 to 8.



Graph. 3. One hour recordings of Cosmic Radiation. Solar flare. Magnetic moment of sunspot on May 21, 1947. Barometer.

June 17.

The increase of cosmic radiation is observed after 9 h. with a maximum at about 11 h. At this time there are the following flares

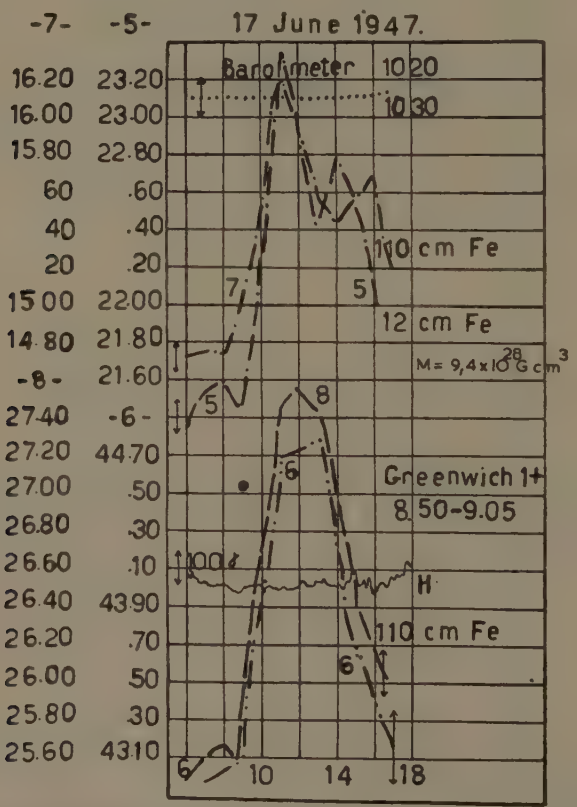
Observation place	Beginning	End	Latitude	Diff. of longitude	Class
Zürich	7 h. 20	7 h. 25	+ 23°	— 52°	point brillant
Wendelstein	8 h. 50		+ 8°	— 9°	1
Greenwich	8 h. 50	9 h. 05	+ 8°	0°	1+

On this day at the latitude of 8° an active region was observed with a longitude of 220° on account of this localisation.

We came to the conclusion that the spot 8626 which is of a bipolar character with a maximum field of 2500 Gausz at 12 o'clock can be the origin of this flare. This spot exists from 12 till 23 June. Here is its history.

8626

Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
June 12, 1947	— 67°	217°	+ 9°	48	4
June 13, 1947	— 51°	219°	+ 9°	97	4
	— 47°	223°	+ 9°	61	1
June 14, 1947	— 37°	221°	+ 8°	73	9
	— 33°	225°	+ 7°	61	1
June 15, 1947	— 25°	220°	+ 8°	24	3
	— 18°	227°	+ 7°	61	2
June 16, 1947	— 12°	220°	+ 9°	36	4
	— 3°	229°	+ 7°	73	1
June 17, 1947	+ 1°	219°	+ 8°	24	9
	+ 9°	227°	+ 7°	48	1
June 18, 1947	+ 15°	220°	+ 10°	73	6
June 19, 1947	+ 28°	220°	+ 9°	73	9
June 20, 1947	+ 40°	219°	+ 9°	218	10
June 21, 1947	+ 54°	219°	+ 9°	291	8
June 22, 1947	+ 65°	217°	+ 10°	194	1
	+ 73°	225°	+ 10°	145	1
June 23, 1947	+ 78°	217°	+ 10°	145	1



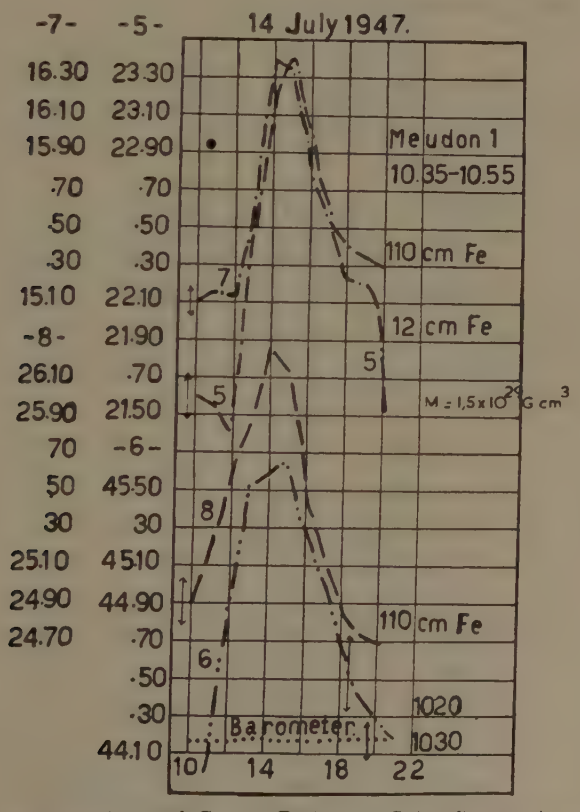
Graph. 4. One hour recordings of Cosmic Radiation. Solar flare and magnetic moment on June 17, 1947. Record of horizontal component of earth's magnetism on that date. Barometer.

July 14.

The increase of intensity of the cosmic radiation begins between 11 and 12 o'clock and the maximum is at about 15 h.

Observed solar flare at Meudon from 10.35—10.55 with a latitude of 13°. In this place on July 14 a spot 8709 is found. The group is unipolar and is followed by a bipolar group. Max. field value H = 1100 Gausz on July 19 at 2000 G.C.T.

8709. Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
July 12, 1947	— 87°	160°	+ 12°	48	1
July 13, 1947	— 74°	160°	+ 12°	194	1
July 14, 1947	— 60°	160°	+ 12°	170	2
July 15, 1947	— 46°	160°	+ 12°	121	5
July 16, 1947	— 32°	162°	+ 12°	73	5
July 17, 1947	— 19°	162°	+ 13°	73	4
July 18, 1947	— 5°	163°	+ 9°	24	2
	— 5°	163°	+ 14°	61	2
July 19, 1947	— 7°	161°	+ 13°	97	4
July 20, 1947	+ 20°	162°	+ 13°	48	2
July 21, 1947	+ 32°	161°	+ 13°	36	1
July 22, 1947	+ 47°	161°	+ 13°	12	2



Graph. 5. One hour recordings of Cosmic Radiation. Solar flare and magnetic moment on July 14, 1947. Barometer.

Physics. — *Solar flares and the origin of Cosmic Radiation.* II. By J. CLAY, H. F. JONGEN and A. J. DIJKER. (Natuurkundig Laboratorium University of Amsterdam.)

(Communicated at the meeting of September 24, 1949.)

July 24.

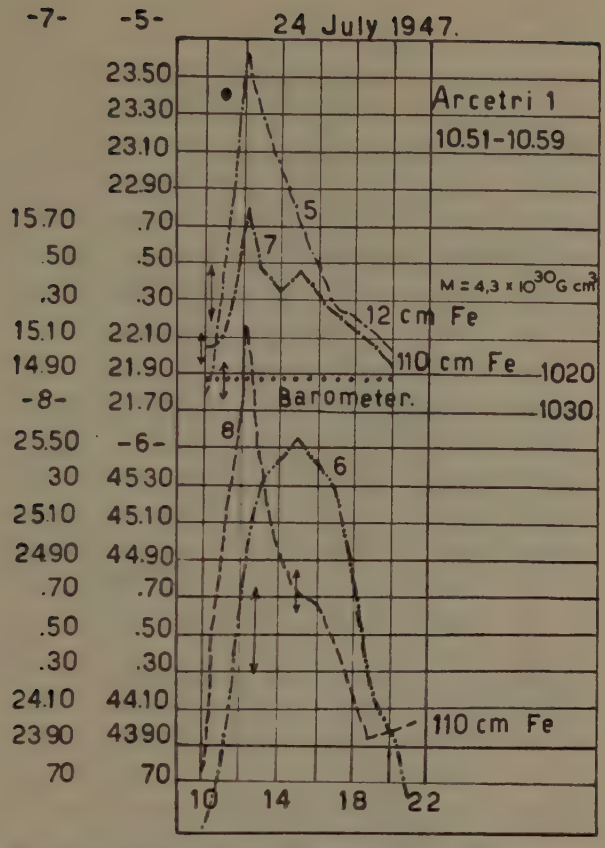
The increase in the radiation occurs after 11 h. with a maximum at about 12 h. A little earlier two flares are observed at Arcetri, namely from 10.51—1059 at latitude +15° and a difference in longitude of —55° of class 1 (deux points brillants) and also from 10.51—10.59 at a latitude of +13° and difference of longitude +82°, class 1 (trois points brillants).

In the surroundings of both these places are the observed spots 8715 and 8707. The first has a maximum field of 1900 Gauss on July 20 at 10 o'clock and 8707 maximum field 4000 Gauss on July 18 at 5 h. 8715 is a dipole and 8707 is an unipole.

The data are following.

8715.

Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
July 14, 1947	— 78°	142°	+ 9°	242	9
July 14, 1947	— 64°	142°	+ 9°	485	12
July 16, 1947	— 55°	139°	+ 11°	436	14
	— 48°	146°	+ 11°	218	12
July 17, 1947	— 41°	140°	+ 11°	582	12
	— 35°	146°	+ 11°	436	11
July 18, 1947	— 28°	140°	+ 11°	630	15
	— 21°	147°	+ 11°	485	10
July 19, 1947	— 15°	139°	+ 12°	630	22
	— 7°	147°	+ 11°	582	19
July 20, 1947	— 2°	140°	+ 12°	388	3
	+ 5°	147°	+ 13°	436	15
July 21, 1947	+ 10°	139°	+ 13°	388	18
	+ 17°	146°	+ 14°	533	35
July 22, 1947	+ 24°	138°	+ 12°	291	36
	+ 33°	147°	+ 14°	485	16
	+ 34°	148°	+ 6°	24	7
	+ 38°	152°	+ 13°	145	1
July 23, 1947	+ 37°	139°	+ 12°	242	12
	+ 46°	148°	+ 13°	533	27
	+ 49°	151°	+ 8°	48	1
July 24, 1947	+ 50°	139°	+ 11°	339	15
	+ 58°	147°	+ 12°	776	18
July 25, 1947	+ 65°	140°	+ 13°	921	11
	+ 73°	148°	+ 13°	630	1
July 26, 1947	+ 79°	140°	+ 13°	970	3

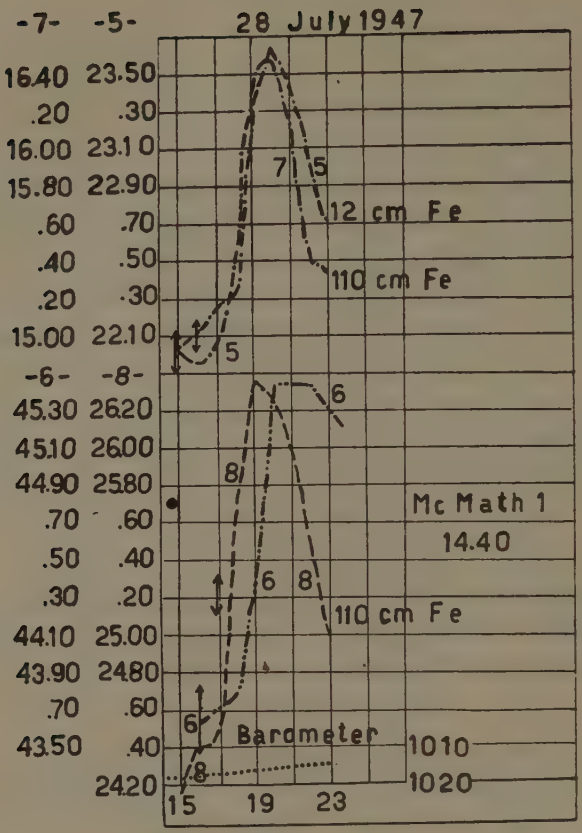


Graph. 6. One hour recordings of Cosmic Radiation. Solar flare and magnetic moment on July 24, 1947. Barometer.

During the period 21—24 July this spot is object to strong changes as apparent from the above table. Presumably the first flare observed by Arcetri belongs to this spot, although there is no satisfactory agreement as to the latitude.

8707. Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
July 11, 1947	— 88°	172°	+ 13°	582	1
July 12, 1947	— 76°	171°	+ 12°	970	2
July 13, 1947	— 61°	173°	+ 10°	121	5
	— 60°	174°	+ 12°	824	1
July 14, 1947	— 49°	171°	+ 12°	679	1
	— 49°	171°	+ 10°	48	6
July 15, 1947	— 34°	172°	+ 12°	724	1
	— 34°	172°	+ 10°	24	6
July 16, 1947	— 19°	175°	+ 12°	679	1
July 17, 1947	— 6°	175°	+ 13°	776	4
July 18, 1947	+ 70°	175°	+ 13°	679	2
July 19, 1947	+ 20°	174°	+ 14°	679	2
July 20, 1947	+ 33°	175°	+ 12°	679	1
July 21, 1947	+ 45°	174°	+ 13°	679	1
July 22, 1947	+ 60°	174°	+ 12°	630	11
July 23, 1947	+ 74°	176°	+ 14°	582	2
July 24, 1947	+ 88°	177°	+ 12°	145	4

On July 24 the spot disappears over the West edge of the solar disc.



Graph. 7. One hour recordings of Cosmic Radiation. Solar flare on July 28, 1947. Barometer.

August 11, 1947.

The increase of intensity sets in after 16 h. G.C.T. and has its maximum value between 17 and 18 h. G.C.T. At about 12 h. G.C.T. 3 flares are observed at MUSWELL HILL and SCHAUINSLAND, but it is not probable that these are the origin of the effect in the Cosmic Radiation vide the great difference in time. No flare is reported during the hours from 12.22—17.05, although during parts of this period several stations have made observations. At the latter moment Mc. Math. observed a flare class 1 at a latitude +10° and a difference in longitude —10° followed on 17 h. 45 by a second flare at a latitude of —20° and a difference of longitude +50°, also of class 1. So the cause for the arising of the increase in intensity is uncertain. It is acceptable however that its continuation after 17 h. is due to the flares mentioned, especially to the first one, as this occurs doubtlessly before 17.05. We will look for spots in the neighbourhood of those two flares.

For the first flare a spot 8767 comes into consideration which is observed from 6—19 of August. 8767 consists of a complex group in which no bipolar parts can be distinguished. Maximum field 2300 Gauss on Aug. 13 at 0 h. G.C.T. The second flare can be correlated with the spot 8764 from Aug. 6—13 after which date it dissappears from the visible part of the sun. Maximum H 1300 Gauss on Aug. 7 at 15 h. The spot was bipolar. The development was the following: (situations every day about 15 h. G.C.T.)

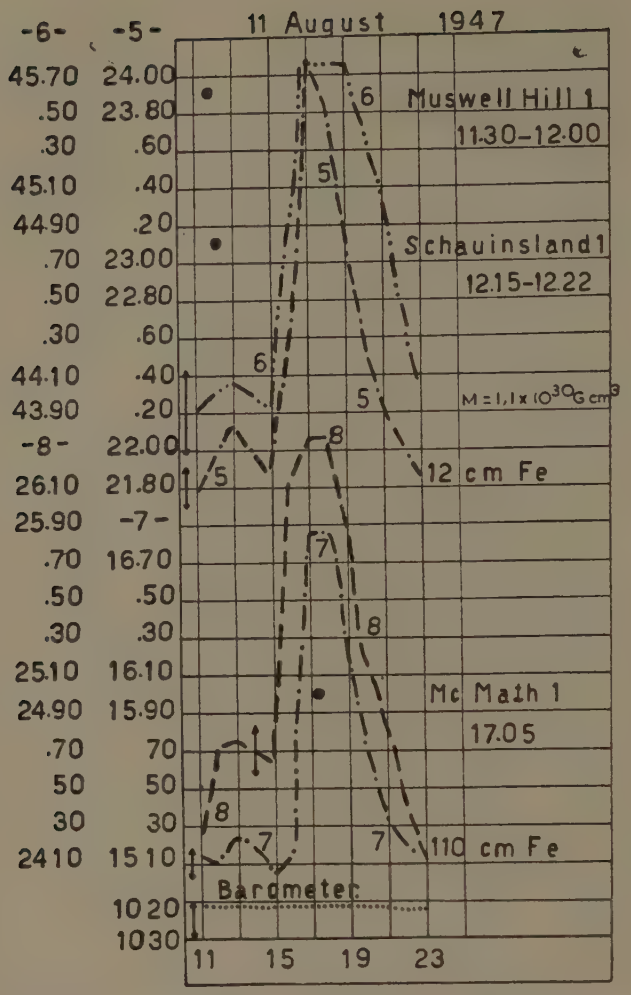
8767.

Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
August 6, 1947	— 67°	210°	+ 10°	12	1
August 7, 1947	— 79°	184°	+ 10°	242	3
	— 69°	194°	+ 10°	24	2
August 8, 1947	— 70°	180°	+ 10°	73	3
	— 64°	186°	+ 9°	242	1
	— 58°	192°	+ 9°	24	1
	— 49°	201°	+ 9°	48	3
August 9, 1947	— 50°	186°	+ 12°	194	12
	— 46°	190°	+ 12°	145	16
	— 36°	200°	+ 10°	97	12
August 10, 1947	— 39°	180°	+ 11°	12	1
	— 28°	191°	+ 12°	776	42
	— 17°	202°	+ 10°	121	9
August 11, 1947	— 18°	192°	+ 10°	1018	53
August 12, 1947	— 19°	178°	+ 7°	24	7
	— 5°	192°	+ 10°	873	47
August 13, 1947	+ 8°	192°	+ 11°	1164	49
	+ 22°	206°	+ 11°	12	1
August 14, 1947	+ 20°	191°	+ 11°	1357	52
	+ 35°	206°	+ 10°	12	1
August 15, 1947	+ 33°	191°	+ 11°	1357	45
August 16, 1947	+ 47°	191°	+ 11°	1406	42
August 17, 1947	+ 60°	191°	+ 12°	436	14
	+ 75°	206°	+ 9°	582	5
August 18, 1947	+ 73°	191°	+ 12°	436	9
	+ 88°	206°	+ 9°	582	3

From 10 to 11 August in this spot there are great changes. This spot dissappears from sight via the west edge of the solar disc.

8764

Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
August 7, 1947	— 1°	262°	— 18°	388	23
August 8, 1947	+ 12°	262°	— 19°	533	12
August 9, 1947	+ 25°	261°	— 19°	315	27
August 10, 1947	+ 39°	258°	— 19°	242	21
	+ 40°	268°	— 19°	242	1
August 11, 1947	+ 48°	258°	— 19°	145	4
	+ 58°	268°	— 20°	242	5
August 12, 1947	+ 61°	258°	— 19°	97	3
	+ 66°	263°	— 19°	12	1
	+ 72°	269°	— 19°	242	1
August 13, 1947	+ 75°	259°	— 19°	121	3



Graph. 8. One hour recordings of Cosmic Radiation. Solar flare and magnetic moment on Aug. 11, 1947. Barometer.

August 15, 1947.

The increase of intensity in cosmic radiation is observed at 10 h. G.C.T.

Solarflares were observed at different places of which we take in consideration the two following:

1°. Observed at WENDELSTEIN from 7.15—7.30 at a latitude +13° and a difference of longitude —12°. Class 1.

2°. Observed at Cambridge at 9.05 G.C.T. at the latitude —10° and a difference of longitude +75°. Class 2.

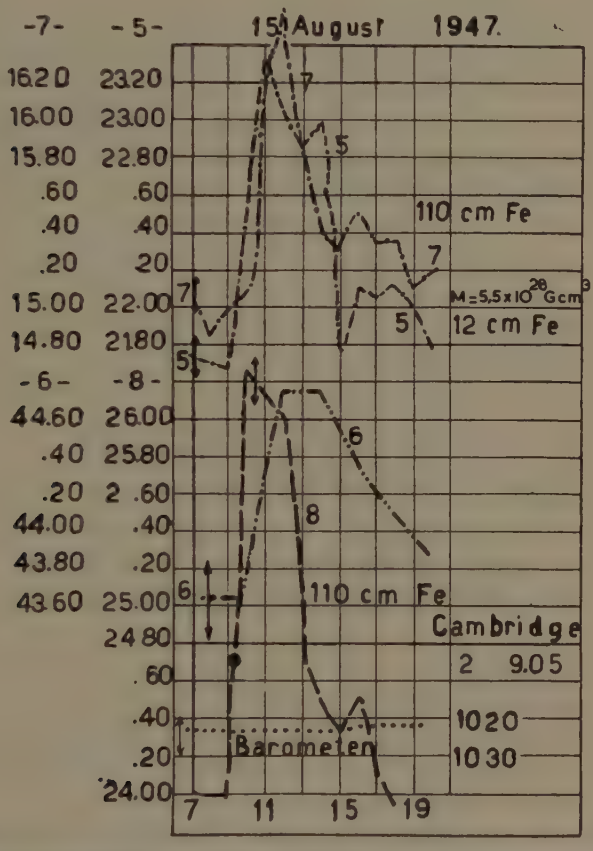
The flare observed at WENDELSTEIN is situated in the surroundings of spot 8774 which has a unipolar character, maximal field 4000 Gauss on Aug. 16 at 2.00 G.C.T.

The Cambridge flare was situated at the same place as spot 8775 which was visible from 12—15 of August after which moment it disappeared over the West edge of the solar disc. The maximum field of 900 Gauss was on Aug. 9 at 10.00 G.C.T. The spot was bipolar.

For the observed effect the flare observed at Cambridge can be considered origin. The development was following.

8775.

Data	Diff. of longitude	Longitude	Latitude	Surface in 10^{-6} of the sunsurf.	Number of smaller parts
August 12, 1947	+ 43°	240°	— 12°	73	3
August 13, 1947	+ 55°	239°	— 9°	36	3
August 14, 1947	+ 70°	241°	— 13°	97	3



Graph. 9. One hour recordings of Cosmic Radiation. Solar flare and magnetic moment on Aug. 15, 1947. Barometer.

In a good number of cases we found at the same time as the flares major variations in the ionosphere which we detected by disturbances in long range radio transmission, but we know that mostly these are produced by photon radiation in contrary to the corpuscular radiation, which must be the origin of the increase of cosmic radiation.

At the present time we have to recognize different possibilities for the excess in the cosmic radiation to originate in the sun's activity. The first theory is that particles are thrown out of the sun by a mechanism somewhat like a synchrotron. The second is that the phenomenon would be caused by the variation of the magnetic fields, as suggested by BAGGE [9])

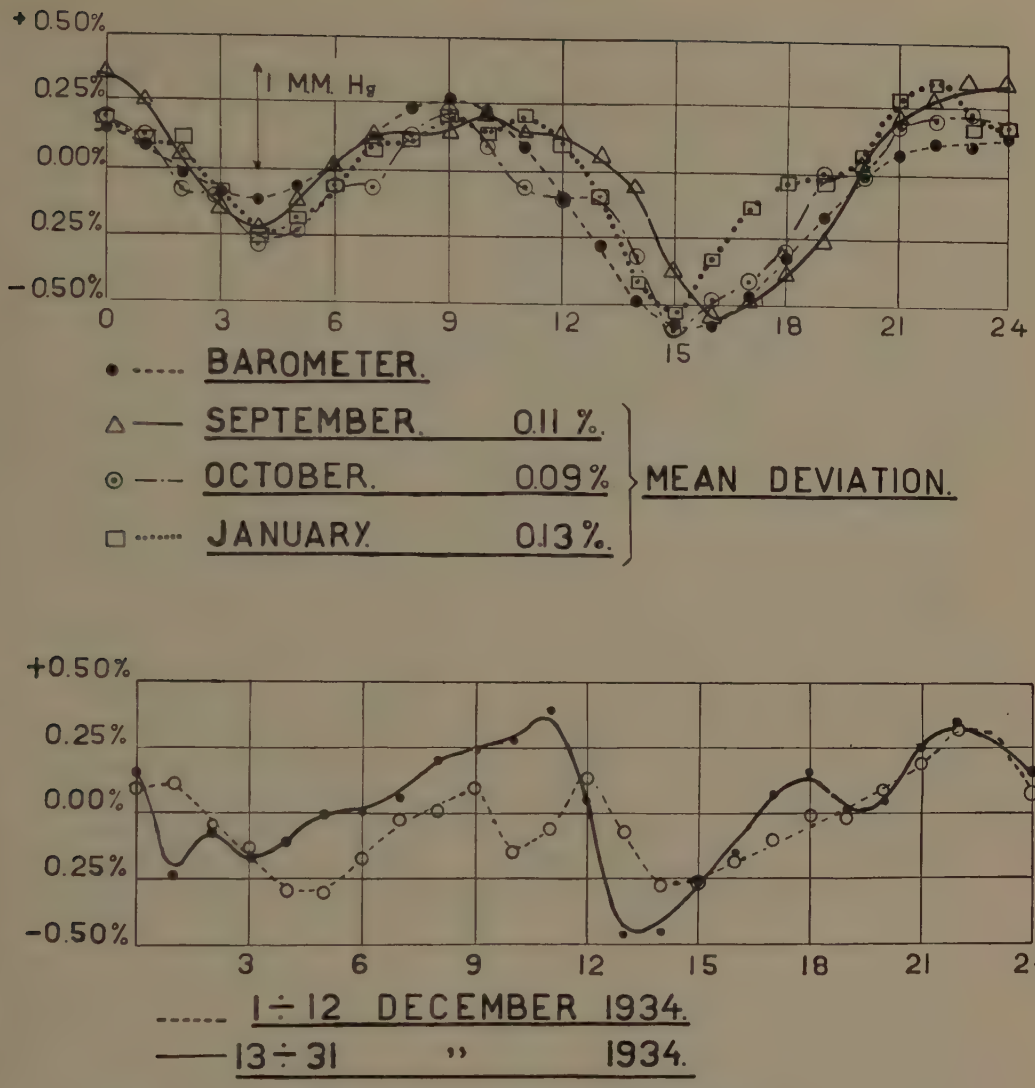
(Como congress Sept. 1949). If it were correct however that the magnetic moment of the sun is 10^{34} Gauss cm^3, it would be certain that particles could not escape from the sun without the presence of a compensating magnetic moment, but in all the cases investigated up till now no indication is found of a moment of that order of magnitude. And in some cases the spots which might be responsible for the phenomenon were even unipolar. Many contemporary astronomers doubt the above given value of 10^{34} on other grounds and they think that at any rate the value has to be variable.

The continuous mean radiation, the instantaneous variations of which are treated of above, may be of solar, galactic or extra-galactic origin. We are of opinion that, if the sun can produce instantaneous flares of cosmic rays, also other stars and especially novae may contribute to the observed radiation. The effect of individual stars must be very much smaller than that from the sun, but as their number is very large, their collective contribution can be pretty well invariable in time. In that case, however, it is a mystery why no directional effect of our galaxy is observable, as we shall point out at the end of this article.

Another theory put forward by ALFVÉN [10]) assumes that small magnetic fields can be produced in the galaxy by the movement of dust clouds and these fields will be able to accelerate particles, as was shown by FERMI [11]). The objection is, that the acceleration takes far too much time. A proton with a minimum energy of 2×10^8 eVolt e.g. when injected into an accelerating field can indeed be accelerated. But to reach an energy sufficient to be registered in cosmic radiation requires 100 million years. ALFVÉN's assumption is, that the origin of the whole of c.r. lies with this accelerating mechanism, but this seems to be in contradiction with the facts stated above about solar flares and their correlation which takes only two hours or less.

For the magnetic latitude of our place of observation the energy necessary to penetrate the magnetic field of the earth is known to be 2×10^9 eV. Another 2×10^9 eVolt are wanted to penetrate the atmosphere. As the relative variation in the open vessel sometimes differed from that in the vessel shielded by 110 cm Fe (the shield taking 10^9 eV to be penetrated by the secondaries, i.e. the mesons) the distribution of energy in the excess radiation must have differed from that in the permanent radiation. We are intending to make a research on the latter subject. To this end we are going to record the flares at different depths in the same coalmine in which we have recorded radiation intensity before. Comparing recordings at different depths, we can get an idea of the distribution of the energies of the incident excess radiation correlated with solar flares.

From a communication by M. SCHEIN [12]) at the Como conference appeared that on May 11th 1949 in Chicago at very great height in the atmosphere an enormous increase of cosmic radiation was measured of about 75 %. At that time there was a solar flare of intensity 3. The only thing we found in our recordings was an increase of 1 % in the open vessel



Graph. 10. CLAY & WOLTJER. Recording of Cosmic Radiation during 4 months at Bandoeng, geomagn. lat. —18°. Barometer.

and in the vessel shielded with 12 cm between 2 and 4 h. G.C.T., but no trace of an increase in the vessels shielded with 110 cm. This particular excess radiation appears therefore to have been of low energy.

Finally we should like to draw attention to the recordings of the ionisation by cosmic radiation considered in relation with sidereal time graph. 10. These recordings do not show any variation with sidereal time exceeding 0.1 %, which is an essential criterion showing that there is no indication of an extra-galactic origin of the radiation nor of a difference with direction in the galactic system.

Our best thanks are due to Dr H. P. TH. VAN LOHUIZEN for his useful help with magnetic and solar data.

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Amsterdam, Sept. 1949.

Botany. — *The respiration of the stem of ripening sugar cane.* By J. W. HES.

(Communicated at the meeting of September 24, 1949.)

Introduction.

The production of the photosynthetic activity of a sugar cane plant provides the greater part of the material for the building up of the plant body and is also a source of energy for both construction (growth) and maintenance. The excess of the carbondioxide assimilation can be stored as a reserve material, in the case of sugar cane sucrose, in such an abundance, that it can be extracted from the stalks on an economic basis and thus cane is grown for this reason.

The difference between the total amounts of carbondioxide assimilation products and the part thereof incorporated in the plant tissues (dry matter) or serving as the source of energy consumed in respiration, is roughly speaking the amount of sugar in the cane at harvest time.

We are fairly well informed about the dry matter after years and years of cropping cane. Concerning assimilation there are some data available from preliminary investigations, for instance those carried out in Java by KAMERLING (4), KUYPER (5, 6) and COELINGH and KONINGSBERGER (2). But as far as we know there is no quantative information about respiration processes, at least not in the Java-literature. Hence we started a series of experiments to obtain some preliminary information on this matter.

Material and Methods.

In working out these experiments cane was chosen of one variety, twelve months old and grown under conditions representing approximately the average Java conditions in the cane fields. The material involved in the experiments was selected from stems of about the same size, length and habitus. After cutting the internodes were cleaned with soap and water, rinsed and dried. To prevent desiccation both ends were dipped in melted paraffine.

The respiration was determined by measuring the carbondioxide production. A current of humid air deprived of carbondioxide was passed through a respiration vessel, having a volume only slightly exceeding that of the cuttings, and then in succession two Pettenkofer-tubes containing barytawater. The carbondioxide liberated in two hours by the internodes was absorbed and determined in the usual manner by titration. All experiments have been carried out in a dark room at 28° C.

In order to find the amount of dry matter (total solids), sucrose and reducing sugars, the internodes used for measuring the carbondioxide pro-

duction were weighed (fresh weight) and then chopped to a finely divided condition and subsequently crushed in a small hydraulic press to disintegrate the tissues. The pulp along with the juice already expressed by this treatment was made up to a known weight by the addition of water. After adjusting the pH to 7.5 and adding one gram of calciumcarbonate the mixture was digested for one hour at boiling temperature, this proved to be sufficient for a complete extraction of all sugars. After cooling the acidity was checked and found to be neutral in all determinations.

In the fiber determination the pulp was separated from the extract and washed in running water until the washings were free from dissolved sugar as disclosed by the α-naphtol reaction. The sugar-free fiber was dried to constant weight in a drying oven at 105 °C (fiber-weight) and then immersed in a 10 % sucrose solution for 12 hours in a refrigerator. Water was imbibed by the fiber, thus leaving the sucrose solution more concentrated. The increase in polarization is a means for computing the amount of "colloid" or "imbibition" water attached to the fiber. We assumed this amount equal to that originally present in the cane tissues (in Dutch: brixvrij rietwater, translated: brix-less cane-water).

The separated extract was analysed by methods usually employed in the Java sugar industry (3). The amount of dissolved substance (gravity solids) was calculated from the pycnometric specific gravity, whereas after clarification reducing sugars were determined by the Schoorl's iodide method with Luff's solution (3). Sucrose determination was carried out ter clarification by double polarization using STEUERWALD's method of inversion at room temperature (7). The results obtained in this way were checked by a chemical sucrose determination calculated from the difference between the invert sugar contents before and after inverting, cf. BROWNE and ZERBAN (1). Both methods gave equal quantities of sucrose.

In order to refer the percentages of substances found in the extract to a fresh weight of cane basis, the following formula was used:

$$\frac{a\left\{b-\left(c+\frac{c\times d}{100}\right)\right\}}{e} = \%\ \text{originally present in the cane, where:}$$

a = percentage of substance (sucrose, reducing sugars or gravity solids) in the extract

b = total of fresh cane weight and added water

c = fiber weight

d = "colloid" or "imbibition" water in percentage of the fiber

e = fresh weight

The total dry weight (total solids) was calculated by adding the fiber weight to the amount of gravity solids reduced to cane basis by the formula given above.

Nitrogen determinations were carried out by the usual Kjeldahlmethod.

Experiments.

After cutting the carbondioxide production turned out to be markedly higher than normal being a reaction to injury. To avoid this effect the cuttings were always prepared 24 hours in advance for all experiments. The carbondioxide production was determined the next day and the day after, generally for two periods of two hours daily. The two daily values agreed very well, the differences never exceeding 10 %. After these two days the cuttings were analysed, thus determining the fresh weight, dry weight and percentage of sucrose and reducing sugars of the stem part involved in an experiment. These data are plotted in the graphs of figure 1 and 2. The horizontal axis, laid off with a centimeter scale, represents the heights of the internodes above the surface of the soil; the schematic drawing of a cane stalk underneath shows the decreasing length of the internodes from foot to top. On the vertical axis the percentage of the constituent parts related to the fresh weight is indicated, with the exception of the nitrogen, which is expressed as a percentage of the dry matter. The total dry weight is indicated by dots, sucrose by circles, reducing sugars by triangles and nitrogen by crosses. Smoothed curves have been drawn through the series of points determined. The number of points indicating the chemical composition is much greater than those in figure 1 denoting the respiration, as in most experiments more than one cutting was used and each was analysed separately. The nitrogen was estimated in internodes chosen along the length of one stalk, the dry matter of these internodes is also introduced in figure 2.

The bottom scale in figure 1 has the same meaning as that in figure 2. On the lefthand scale the production of carbondioxide in the two-hour periods of the first day is indicated in grams per 24 hours per kilogram of dry weight.

Discussion.

The distribution of sugars along the cane stalk as indicated in fig. 1 is perfectly similar to the figures in WENT's publication on this subject (8). WENT also examined the part of the stalk below the soil surface, which has lower sucrose contents, this part was however not involved in our experiments. The remarkable fact that the reducing sugars reach a maximum percentage some internodes below the top is also clearly demonstrated in our graph.

To make sure that neither appreciable gas production nor consumption occurred even during a whole day, we examined internodes of various ages in a one liter flask, closed with a manometer and placed in a bath of constant temperature. Therefore we can assume that sugars, anyhow carbohydrates, are the substrate of the respiration of the cane stalk tissues.

The respiration in the lower part of the stalk up to about one meter, in which the amount of dry matter, sucrose, reducing sugars and nitrogen

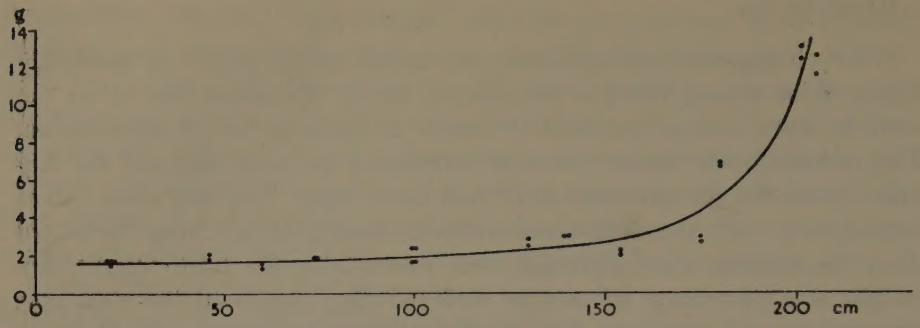

Fig. 1. The march of the respiration level along the stem of ripening sugar cane. The ordinate shows the respiration level expressed as grams of carbondioxide daily produced by 1 kilogram of dry matter, the abscisse the distance from the soil surface.

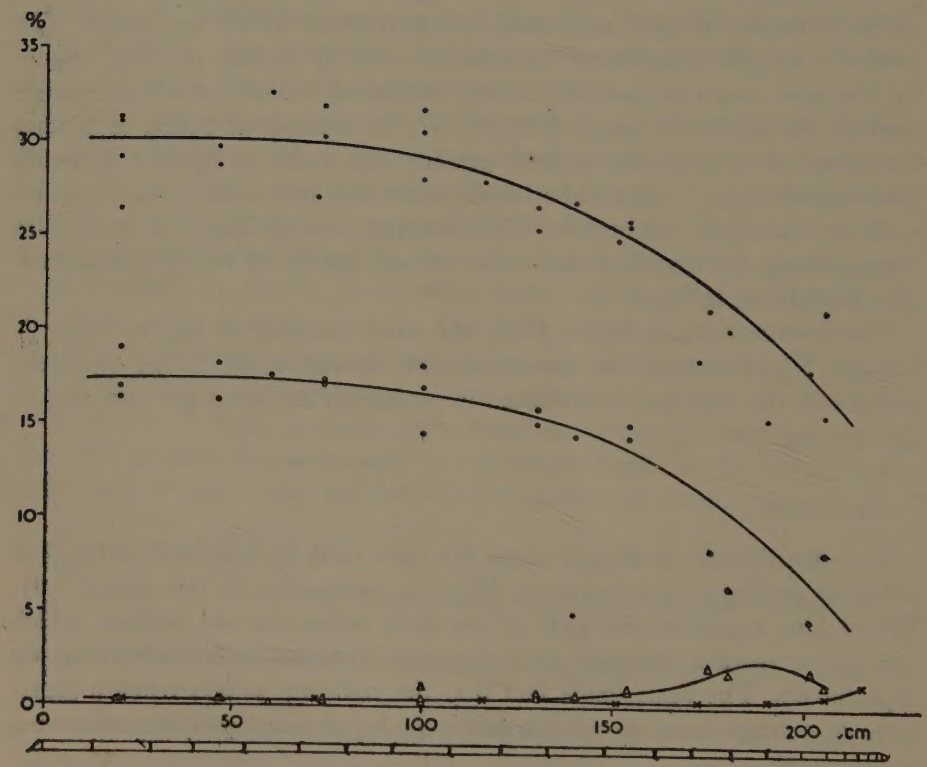

Fig. 2. The chemical composition of a ripening cane stem indicated on the ordinate in % of the fresh weight, with exception of the nitrogen, calculated as % of the dry weight. Sucrose indicated by circles, reducing sugars by triangles, dry matter by dots and nitrogen by crosses. Abscisse as in figure 1.

remains on the same level, has a value of about 2 grams of carbondioxide per 24 hours per kilogram dry matter, corresponding to the dissimilation of about 1.36 grams of hexoses. This part of the stem can be considered as mature and its sucrose contents as at the maximum percentage for the cane under examination. In the next part of the stem the accumulation of sucrose is continuing and the respiration value increases slowly in apical direction with the decreasing sucrose concentration. Still nearer to the top where the growth is gradually becoming more important, the respiration increases rather rapidly to a value of 13 grams of carbondioxide or expressed in terms of the dissimilation of hexoses about 9 grams per kg dry weight per day.

In the younger internodes the percentages of reducing sugars and nitrogen are higher than in the older ones. There is however no direct relation between these quantities and the respiration level. Regarding as dry weight the value obtained by substracting the total sugars from the total dry weight, there is no important fluctuation in this value along the stalk. Hence there is no obvious relation between either this dry weight value or the respiration level. Among the data considered in this investigation the percentage of sucrose is perhaps the best expression for the physiological activity of the internodes along the stem.

I am much indebted to Messrs. P. J. KLOKKERS and W. H. JANSEN for their kind help in carrying out the elaborate analysis and to Mr. C. J. G. ALTMANN for his enthusiastic assistance in executing the respiration experiments and preparing this publication. Without their help it would have been hardly possible to accomplish this investigation.

Summary.

The respiration intensity of ripening sugar cane was examined and measured as carbondioxide production. The lower part of the stalk, which has already attained its maximum sucrose contents up to where sucrose accumulation is still continuing, shows a respiration level of approximately 2 grams of carbondioxide per kilogram daily. From this point upwards the production increases gradually, but near the top there is a sudden increase to approximately 13 grams of carbondioxide. Obviously there is no clear and simple relation between the carbondioxide elimination and the chemical composition as far as sucrose, reducing sugars, nitrogen or dry substance are concerned.

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